Trajectory control of a quadrotor carrying a cable-suspended load

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***Abstract*—In this paper, the problem of controlling the trajectory motion of a quadrotor with a suspended load is considered. Using the Lagrange equations of the second kind, the equations of motion of the quadrotor-load system in three-dimensional space are obtained. The authors propose a control algorithm based on stable differential equations describing transients according to the mismatch of current and target coordinates. The effectiveness of the algorithm is confirmed by the results of numerical modeling.**

***Index Terms*—quadrotor, trajectory control, payload transportation on suspension, desired transition processes**

I. Introduction

Multicopter unmanned aerial vehicles (UAV) are successfully used today in different tasks such as aerial photography and digital mapping [1]. Promising industries for multicopter applying are cargo delivery [2], [3], modular construction, items collection [4], scientific research of some area and precision farming [5].

The problem of cargo transportation using cable suspension is one of the most in demand in these applications. The removal of a payload at a distance equal to the suspension length can significantly reduce the amount of external interference from control signals and incoming air flows from the propellers of the quadrotor.

Suspension oscillations arising during maneuvering significantly change the UAV's flight characteristics and can lead to unsatisfactory or dangerous flight conditions. Moreover, in some applications, such as aeromagnetic surveys, it is necessary to ensure the movement of the payload — magnetometer with subdecimeter accuracy along the prescribed route [6], [7], [8]. In this problem formulation the synthesized control laws should provide suppression of the suspension oscillations relative to the vertical and stabilization of the load on a given trajectory.

The synthesis of control laws for a dynamic quadrocopter-cargo system requires the use of the modern theory of nonlinear automatic control systems.

Combined controller $H_2/H\infty$ is proposed for solving trajectory control tasks in [9]. Methods of geometric control theory are used to synthesize control laws of quadrotors group co-transporting cargo [10], [11] and to control a quadrotor with a suspension of several links [12]. Numerical optimization algorithms for constructing a motion path are proposed in [13]. Open-loop control system based on the dynamic programming method is proposed in [6].

In the above papers generally only global asymptotic convergence of the trajectory control error is proved, and the task of calculating the control actions providing a given form of transient processes in a closed-loop system remains relevant.

This work is a development of [14], in which the problem of motion controlling of the quadrotor-cargo system was solved in one plane. The dynamics of the system in three-dimensional space is obtained.

The synthesis of control signals is based on the required differential equations of the controlled parameters behavior in time and a complete dynamics model. The results of numerical simulation confirm the effectiveness of the proposed control method.

II. Problem Statement

In this study, the system is consisted of quadrotor M_1 and load M_2 whose centers of masses were connected via weightless cable with length l Fig. 1. We introduced the angle between vertical and cable through two projections γ-angle in (x,z) plane and ζ-angle in (y,z). Orientation of

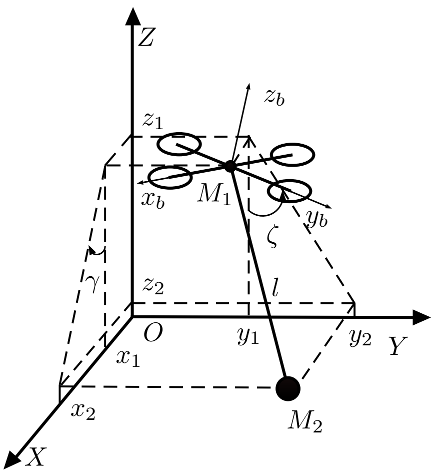

Figure 1. Quadrotor with a payload on suspension.

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quadrotor-connected coordinate system x_b, y_b, z_b towards fixed coordinate system was determined by Euler angles ψ, ϕ, θ. It is sufficient to use any two Euler angles to control the quadrotor motion direction, so further we considered the system with $\psi = 0$. Coordinates of rigid body x_2, y_2, z_2 were expressed through coordinates of quadrotor x_1, y_1, z_1, length of cable l and angles γ, ζ:

$$\begin{cases} x_2 = x_1 + lR(\gamma,\zeta)A(\gamma,\zeta), \\ y_2 = y_1 + lR(\gamma,\zeta)B(\gamma,\zeta), \\ z_2 = z_1 - lR(\gamma,\zeta)C(\gamma,\zeta), \end{cases} \tag{1}$$

where

$$R(\gamma,\zeta) = 1/\sqrt{1 - D(\gamma,\zeta)^2} \tag{2}$$

and

$$\begin{aligned} A(\gamma,\zeta) = \sin\gamma\cos\zeta; B(\gamma,\zeta) = \cos\gamma\sin\zeta; \\ C(\gamma,\zeta) = \cos\gamma\cos\zeta; D(\gamma,\zeta) = \sin\gamma\sin\zeta. \end{aligned} \tag{3}$$

To calculate equations of motion for all components, we used the extended Hamiltonian principle connecting derivative of the Lagrangian with generalized coordinates $\boldsymbol{q}$ and generalized forces $\boldsymbol{Q}$:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{Q}, \tag{4}$$

$$\boldsymbol{q} = \left[\ x_1, y_1, z_1, \theta, \phi, \gamma, \zeta\ \right]^T, \tag{5}$$

$$\boldsymbol{Q} = \begin{bmatrix} u_1\sin\theta\cos\phi, \\ -u_1\sin\theta, \\ u_1\cos\theta\cos\phi, \\ u_2, \\ u_3, \\ 0, \\ 0 \end{bmatrix}. \tag{6}$$

Here u_1 — total thrust; u_2, u_3 — torque moments.

Lagrangian of the system with (1):

$$\begin{aligned} L =& \frac{1}{2}m_1\left(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2\right) + \frac{1}{2}I_{yy}\dot{\theta}^2 + \frac{1}{2}I_{xx}\dot{\phi}^2 + \\ &+ \frac{1}{2}m_2\left(\left(\dot{x}_1 + l\left(\dot{\gamma}C - \dot{\zeta}D\cos^2\gamma\right)R^3\right)^2 + \right. \\ &+ \left(\dot{y}_1 + l\left(\dot{\zeta}C - \dot{\gamma}D\cos^2\zeta\right)R^3\right)^2 + \\ &+ \left.\left(\dot{z}_1 + l\left(\dot{\zeta}B\cos^2\gamma + \dot{\gamma}A\cos^2\zeta\right)R^3\right)^2\right) - \\ &- m_1gz_1 - m_2g\left(z_1 - lCR\right), \end{aligned} \tag{7}$$

where m_1, m_2 denote quadrotor M_1 and load M_2 mass respectively; g is acceleration of gravity; I_{yy} and I_{xx} are quadrotor inertia relative to the transverse axis of rotation passing through the c.o.m. of mass of the quadrotor.

Finally, from these equations we derived equations of motion for each component of translational motion:

$$\begin{cases} m\ddot{x}_1 = u_1\sin\theta\cos\phi - lm_2\left(A\ddot{R} + \left(2C\dot{\gamma} - 2\dot{\zeta}D\right)\dot{R} + \right. \\ \qquad \left. + \left(C\ddot{\gamma} - A\dot{\gamma}^2 - 2\dot{\zeta}\dot{\gamma}B - D\ddot{\zeta} - A\dot{\zeta}^2\right)R\right), \\ m\ddot{y}_1 = -u_1\sin\phi - lm_2\left(B\ddot{R} + \left(2C\dot{\zeta} - 2\dot{\gamma}D\right)\dot{R} + \right. \\ \qquad \left. + \left(C\ddot{\zeta} - B\dot{\zeta}^2 - 2\dot{\zeta}\dot{\gamma}A - D\ddot{\gamma} - B\dot{\gamma}^2\right)R\right), \\ m\ddot{z}_1 = u_1\cos\phi\cos\theta + lm_2\left(C\ddot{R} + \left(-2A\dot{\gamma} - 2\dot{\zeta}B\right)\dot{R} + \right. \\ \qquad \left. + \left(-B\ddot{\zeta} - C\dot{\zeta}^2 + 2\dot{\zeta}\dot{\gamma}D - A\ddot{\gamma} - C\dot{\gamma}^2\right)R\right) - mg \end{cases} \tag{8}$$

and rotational motion:

$$\begin{cases} \ddot{\zeta}\left(A^2 - 1\right)lR = -l\left(DA\ddot{R} + \left(2CD\dot{\gamma} + 2\dot{\zeta}A^2 - 2\dot{\zeta}\right)\dot{R} + \right. \\ \qquad \left. + \left(CD\ddot{\gamma} - DA\dot{\zeta}^2 + 2\dot{\zeta}\dot{\gamma}CA - DA\dot{\gamma}^2\right)R\right) - \\ \qquad - \ddot{x}_1D + \left(\ddot{z}_1 + g\right)B + \ddot{y}_1C, \\ \ddot{\gamma}\left(B^2 - 1\right)lR = l\left(BD\ddot{R} + \left(2CD\dot{\zeta} + 2\dot{\gamma}B^2 - 2\dot{\gamma}\right)\dot{R} + \right. \\ \qquad \left. + \left(CD\ddot{\zeta} - DB\dot{\zeta}^2 + 2\dot{\zeta}\dot{\gamma}CB - BD\dot{\gamma}^2\right)R\right) + \\ \qquad + \ddot{x}_1C + \left(\ddot{z}_1 + g\right)A - \ddot{y}_1D, \\ I_{xx}\ddot{\phi} = u_2, \\ I_{yy}\ddot{\theta} = u_3. \end{cases} \tag{9}$$

Here $m = m_1 + m_2$.

In this research the task was to create a control algorithm for load motion. Controller initially brought center of mass of the load to the target position x_{ref}, y_{ref} and z_{ref} coordinates and further provided following the position with simultaneous oscillation suppression. We needed to find appropriate control actions u_1, u_2, u_3, which provided decreasing to zero difference between current and target coordinates and decreasing to zero suspension deviation angles from vertical γ, ζ. It was assumed we knew coordinates of c.o.m. of quadrotor, its spatial orientation, angles γ, ζ and their derivatives in each moment of time. The rotors of the quadrotor had enough power.

III. Control algorithm synthesis

To find the control actions u_1, u_2, u_3 that solve the above problem, we choose the desired motion trajectory of the S center of mass of the load to the target position and the suspension deviation angles to the vertical position

$$S : (x_2, y_2, z_2, \gamma, \zeta) \to (x_{ref}, y_{ref}, z_{ref}, 0, 0). \tag{10}$$

Let us define the functions:

$$\begin{cases} S_x = \dot{x}_2 + k_x\left(x_2 - x_{ref}\right), \\ S_y = \dot{y}_2 + k_y\left(y_2 - y_{ref}\right), \\ S_z = \dot{z}_2 + k_z\left(z_2 - z_{ref}\right), \\ S_\gamma = \dot{\gamma} + k_\gamma\gamma, \\ S_\zeta = \dot{\zeta} + k_\zeta\zeta, \end{cases} \tag{11}$$

and demand the fulfillment of the conditions

$$S : S_x = S_y = S_z = S_\gamma = S_\zeta = 0. \tag{12}$$

Transients in the variables $x_2, y_2, z_2, \gamma, \zeta$ will be given by differential equations system (11), (12).

Realization of conditions (12) for finding the desired trajectory guarantees the aperiodic movement of the load and suspension to a predetermined position. At a certain point in time, variables $x_2, y_2, z_2, \gamma, \zeta$ may not satisfy the equations (12). It is necessary to change the control actions u_1, u_2, u_3 to bring the system to the trajectory (12) and hold it on this trajectory until the transition to a predetermined position. To do this, we require the realization of the conditions

$$\begin{aligned} &\dot{S}_x = -\alpha_x S_x; \dot{S}_y = -\alpha_y S_y; \dot{S}_z = -\alpha_z S_z;\\ &\dot{S}_\gamma = -\alpha_\gamma S_\gamma; \dot{S}_\zeta = -\alpha_\zeta S_\zeta. \end{aligned} \tag{13}$$

In the equations (11) and (13) k_x, k_y, k_z, k_γ, k_ζ and α_x, α_y, α_z, α_γ, α_ζ — are positive constant coefficients that determine the time the cargo goes to the target position. Equations (13) describe the stable processes of the system movements to a given position. Substituting (11) in (13), we get

$$\begin{cases} \ddot{x}_2 + k_x \dot{x}_2 = -\alpha_x \dot{x}_2 - \alpha_x k_x (x_2 - x_{ref}),\\ \ddot{y}_2 + k_y \dot{y}_2 = -\alpha_y \dot{y}_2 - \alpha_y k_y (y_2 - y_{ref}),\\ \ddot{z}_2 + k_z \dot{z}_2 = -\alpha_z \dot{z}_2 - \alpha_z k_z (z_2 - z_{ref}),\\ \ddot{\gamma} + k_\gamma \dot{\gamma} = -\alpha_\gamma \dot{\gamma} - k_\gamma \alpha_\gamma \gamma,\\ \ddot{\zeta} + k_\zeta \dot{\zeta} = -\alpha_\zeta \dot{\zeta} - k_\zeta \alpha_\zeta \zeta. \end{cases} \tag{14}$$

Considering

$$\begin{cases} \dot{x}_2 = \dot{x}_1 + lR^3 \left(\dot{\gamma} C - \dot{\zeta} D \cos^2 \gamma \right),\\ \dot{y}_2 = \dot{y}_1 + lR^3 \left(\dot{\zeta} C - \dot{\gamma} D \cos^2 \zeta \right),\\ \dot{z}_2 = \dot{z}_1 + lR^3 \left(\dot{\zeta} B \cos^2 \gamma + \dot{\gamma} A \cos^2 \zeta \right) \end{cases} \tag{15}$$

and

$$\begin{cases} \ddot{x}_2 = \ddot{x}_1 + l \Big(A\ddot{R} + 2 \left(C\dot{\gamma} - D\dot{\zeta} \right) \dot{R} +\\ \quad + \left(-D\ddot{\zeta} - A\dot{\zeta}^2 - 2B\dot{\gamma}\dot{\zeta} + \ddot{\gamma} C - \dot{\gamma}^2 A \right) R \Big),\\ \ddot{y}_2 = \ddot{y}_1 + l \Big(B\ddot{R} + 2 \left(C\dot{\zeta} - D\dot{\gamma} \right) \dot{R} +\\ \quad + \left(-D\ddot{\gamma} - B\dot{\gamma}^2 - 2A\dot{\gamma}\dot{\zeta} + \ddot{\zeta} C - \dot{\zeta}^2 B \right) R \Big),\\ \ddot{z}_2 = \ddot{z}_1 + l \Big(-C\ddot{R} + 2 \left(B\dot{\zeta} + A\dot{\gamma} \right) \dot{R} +\\ \quad + \left(B\ddot{\zeta} + C\dot{\zeta}^2 - 2D\dot{\gamma}\dot{\zeta} + A\ddot{\gamma} + C\dot{\gamma}^2 \right) R \Big), \end{cases} \tag{16}$$

we rewrite the equations (8) relative to the coordinates of the center of mass of the load

$$\begin{cases} m\ddot{x}_2 = u_1 \sin\theta \cos\phi + lm_1 \Big(A\ddot{R} + \left(2C\dot{\gamma} - 2\dot{\zeta} D \right) \dot{R} +\\ \quad + \left(C\ddot{\gamma} - A\dot{\gamma}^2 - 2\dot{\zeta}\dot{\gamma} B - D\ddot{\zeta} - A\dot{\zeta}^2 \right) R \Big)\\ m\ddot{y}_2 = -u_1 \sin\phi + lm_1 \Big(B\ddot{R} + \left(2C\dot{\zeta} - 2\dot{\gamma} D \right) \dot{R} +\\ \quad + \left(C\ddot{\zeta} - B\dot{\zeta}^2 - 2\dot{\zeta}\dot{\gamma} A - D\ddot{\gamma} - B\dot{\gamma}^2 \right) R \Big)\\ m\ddot{z}_2 = u_1 \cos\phi \cos\theta + lm_1 \Big(-C\ddot{R} + \left(A\dot{\gamma} + \dot{\zeta} B \right) 2\dot{R} +\\ \quad + \left(B\ddot{\zeta} + C\dot{\zeta}^2 - 2\dot{\gamma}\dot{\zeta} D + A\ddot{\gamma} + C\dot{\gamma}^2 \right) R \Big) - mg. \end{cases} \tag{17}$$

We introduce the notation for the components of the right-hand sides of the equations (17) containing the factor lm_1 (H_x, H_y, H_z, respectively).

Further, in order to express the necessary control signals taking into account the required differential equations (11), we leave all the components with u_1 on the left side and exclude the second derivatives of the load coordinates using the relations (14):

$$\begin{cases} u_1 \sin\theta \cos\phi = m \left(-\left(\alpha_x + k_x \right) \dot{x}_2 - \alpha_x k_x \left(x_2 - x_{ref} \right) \right) -\\ \quad - H_x,\\ -u_1 \sin\phi = m \left(-\left(\alpha_y + k_y \right) \dot{y}_2 - \alpha_y k_y \left(y_2 - y_{ref} \right) \right) -\\ \quad - H_y,\\ u_1 \cos\theta \cos\phi = m \left(-\left(\alpha_z + k_z \right) \dot{z}_2 - \alpha_z k_z \left(z_2 - z_{ref} \right) \right) -\\ \quad - H_z + mg. \end{cases} \tag{18}$$

We introduce another replacement - we denote the right parts as (18) H_{xx}, H_{yy}, H_{zz} respectively. When squaring the left and right sides of the obtained equations and folding them term by term, we obtain the required total engine thrust

$$u_1 = m\sqrt{H_{xx}^2 + H_{yy}^2 + H_{zz}^2}. \tag{19}$$

From the equations (18) we find the necessary pitch and roll angles that ensure movement along the trajectory (12)

$$\begin{cases} \theta_{ref} = \arctan\left(H_{xx}/H_{zz} \right),\\ \phi_{ref} = \arctan\left(-H_{yy}/\sqrt{H_{xx}^2 + H_{zz}^2} \right). \end{cases} \tag{20}$$

To calculate the control moments u_2, u_3 let us repeat the above procedure for the variables ϕ, θ and obtain controls that implement the desired quadrotor orientation [15]

$$\begin{cases} u_2 = I_{xx}(-(\alpha_\phi + k_\phi)\dot{\phi} - \alpha_\phi k_\phi (\phi - \phi_{ref})),\\ u_3 = I_{yy}(-(\alpha_\theta + k_\theta)\dot{\theta} - \alpha_\theta k_\theta (\theta - \theta_{ref})). \end{cases} \tag{21}$$

The resulting algorithm (18)-(21) is denoted as full. The case of constructing a control algorithm without taking into account the presence of load on the suspension denoted as partial. For the partial algorithm in relations (18) there are no components H_x, H_y, H_z and the coordinates of the center of mass of the quadrocopter x_1, y_1, z_1 are used. Further, the comparison of the full control algorithm with the partial control algorithm is performed.

This comparison allows to evaluate the effect of introducing feedback on the variables γ, ζ into the algorithm for controlling the trajectory motion of the quadrotor-load system.

IV. MODELLING RESULTS

The obtained equations of motion of the quadrotor-load system and the controller are implemented in the Matlab/Simulink environment for simulation. We set the following parameters of quadrotor, suspension and load: $m_1 = 0.4$ kg, $m_2 = 0.05$ kg, $l = 1$ m, $I_{xx} = I_{yy} = 0.0464\ kg \cdot m^2$. The following parameters are set for the control system: $k_x = \alpha_x = 4.0$, $k_y = \alpha_y = 4.0$, $k_z = \alpha_z = 4.0$, $k_\gamma = \alpha_\gamma = 6.0$, $k_\theta = \alpha_\theta = 16.0$, $k_\zeta = \alpha_\zeta = 6.0$, $k_\phi = \alpha_\phi = 16.0$.

From the starting point $z_0 = 1.5$ m, $x_0 = -0.2$ m, $y_0 = -0.2$ m quadrotor with a load began to move along a given piecewise linear path with a constant speed $V = (\dot{x}_{ref}^2 + \dot{y}_{ref}^2)^{0.5} = 0.4$ m/s. The height was set constant $z_{ref} = 1.5$ m for the case of partial regulator and $z_{ref} = 0.5$ m for the case of full regulator.

The trajectories of the cargo, the quadrocopter and the given trajectory in the plane (x, y) are shown in Fig. 2, Fig. 3.

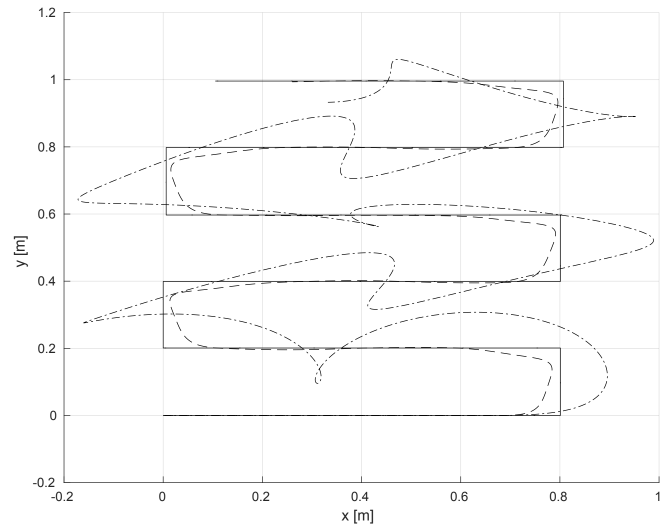

Figure 2. Trajectory of quadrotors motion (dashed), load (dotted) and referenced trajectory (solid) with partial controller

Controlling the quadrocopter-cargo system by means of the partial controller provides more accurate movement of the center of mass of the quadrocopter relative to the given trajectory, Fig. 2, However, the trajectory of the center of mass of the load has a large amplitude of deviation from the given trajectory and curvature compared to the case of control by the full controller, Fig. 3.

The suppression of the suspension vibrations is provided by changing the angles of pitch, roll and total thrust of the quadrocopter engines, that leads to a slight change in flight altitude, Fig. 4.

The full regulator provides half-oscillation and 20% amplitude reduction in position error, Fig. 5. The amplitude of the suspension deviation from the vertical, determined by

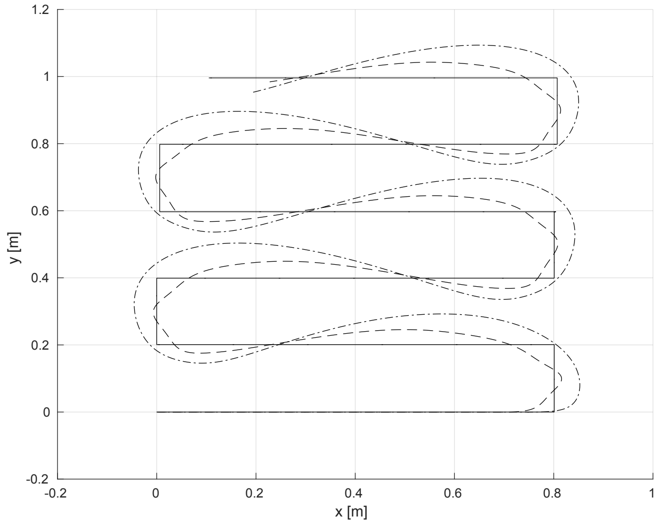

Figure 3. Trajectory of quadrotors motion (dashed), load (dotted) and referenced trajectory (solid) with full controller

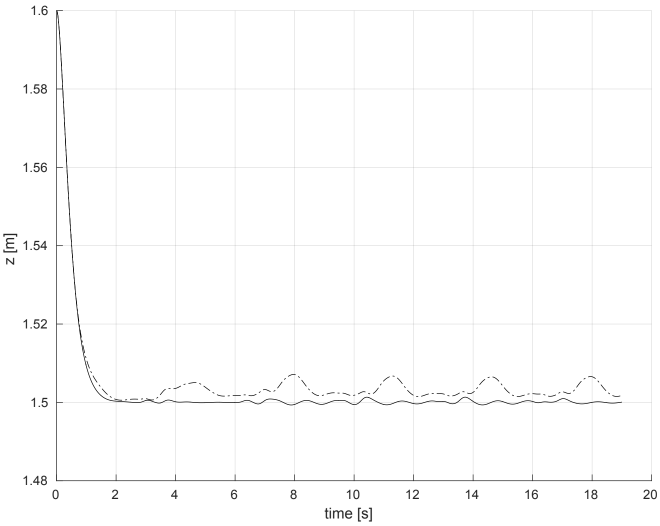

Figure 4. Quadrotor altitude change with full controller (dotted) and with partial controller (solid)

the angle γ, decreases by half when using the proposed control algorithm, Fig. 6. The decrease in the amplitude of the deviation determined by the angle ζ is not so significant due to the absence of a change in the direction of movement along the axis OY, Fig. 7.

V. CONCLUSION

In this study, a nonlinear controller is proposed to navigate a quadrotor-cargo system on a reference position. This nonlinear control takes into account the recent states of a quadrotor with payload on suspension and tries to suppress the suspension deviation from vertical during payload movement along the reference trajectory. This behavior is realized by designing the algorithm based on the required differential equations of transients of controlled variables. The simulation results show that the proposed control system reduces the oscillation of

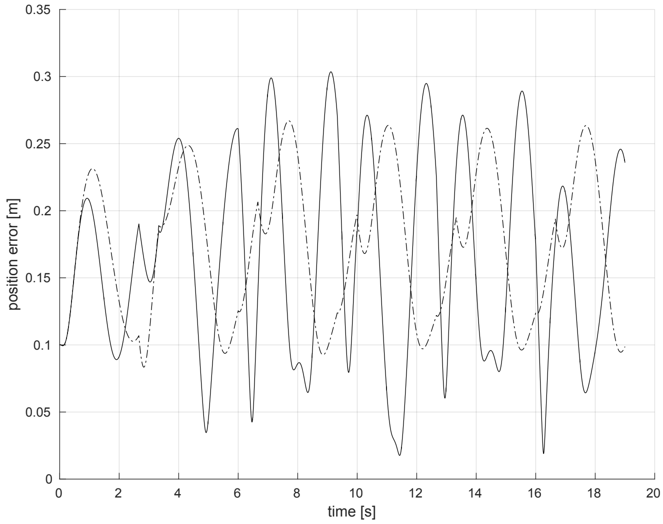

Figure 5. Path control error change with full controller (dotted) and with partial controller (solid)

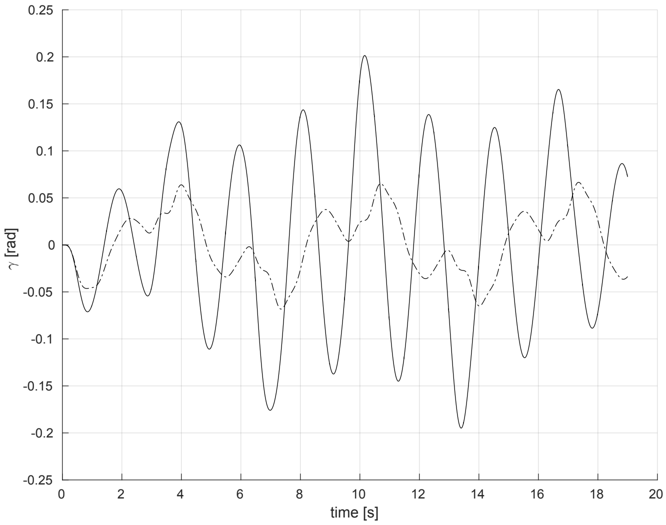

Figure 6. Angle γ oscillations with full controller (dotted) and with partial controller (solid)

the suspension around vertical effectively, compared with the quadrotor position control system that does not include the dynamics of payload on suspension. In the future, we would like to obtain a practical results in indoor flying test on the laboratory bench equipment.

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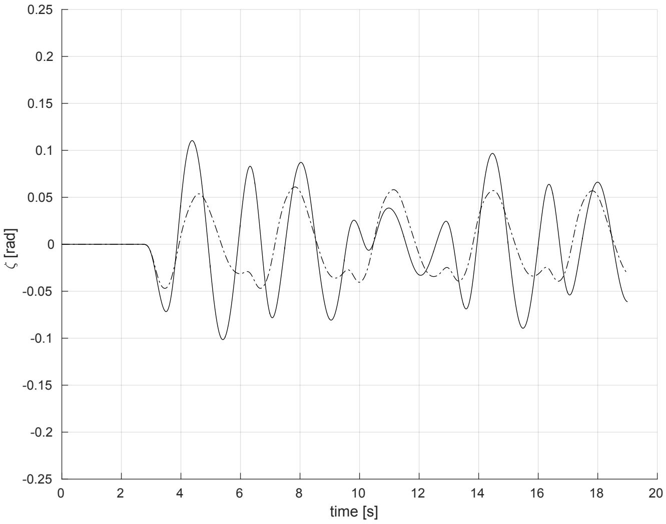

Figure 7. Angle ζ oscillations with full controller (dotted) and with partial controller (solid)

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