Spectral and statistical properties of photoemissions from atomic ensembles in a cat-state field

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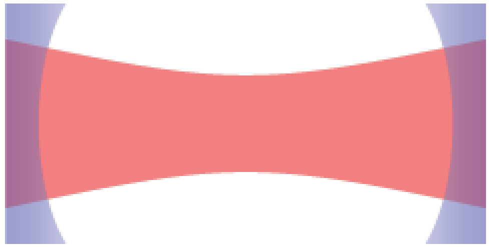
Outline

- Description of the field state
- One atom case: resonance fluorescence spectrum
- Many-atom case: 2^{nd}-order correlation function.

1. Cat-state field

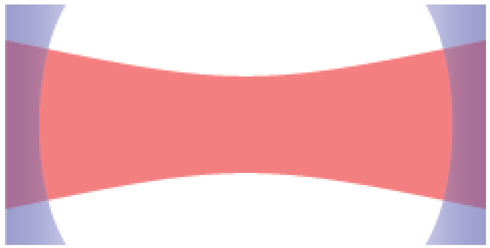


State of the field: $|\alpha\rangle_{YS}$ - Yurke-Stoler state (*B. Yurke, D. Stoler, PRL* ***57****, 13 (1986)*)

$$|\alpha\rangle_{YS} = \frac{1}{\sqrt{2}}\left(|i\alpha\rangle_G + i|-i\alpha\rangle_G\right)$$

$|\alpha\rangle_G$ - Glauber coherent state

1. Cat-state field



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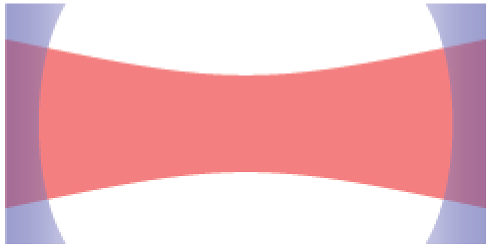
Definition through YS-operators:

$$\hat{a}_{YS}|\alpha\rangle_{YS} = \alpha|\alpha\rangle_{YS}$$

$$\hat{a}_{YS} = e^{\imath\pi\hat{n}}\hat{a}_G \quad \hat{a}^{\dagger}_{YS} = \hat{a}^{\dagger}_G e^{-\imath\pi\hat{n}}$$

1. Cat-state field



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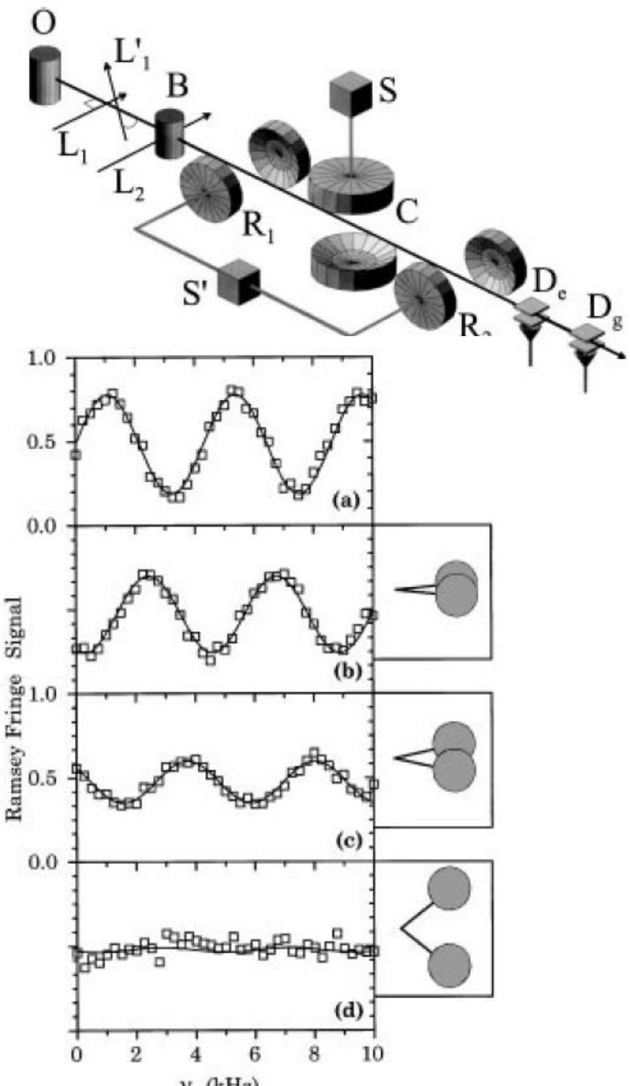
<u>Problem with cat-states - rapid decoherence under photon loss</u>

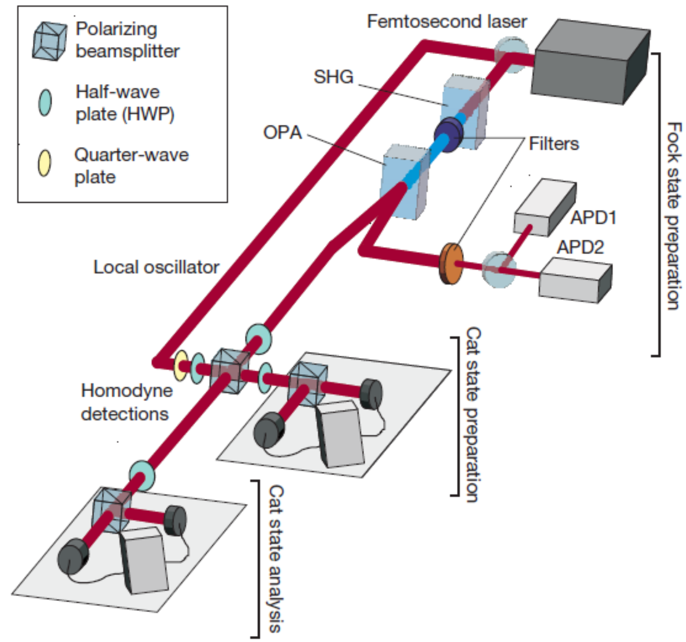
$\Downarrow$

<u>Need field recreation mechanism for steady-state interactions</u>

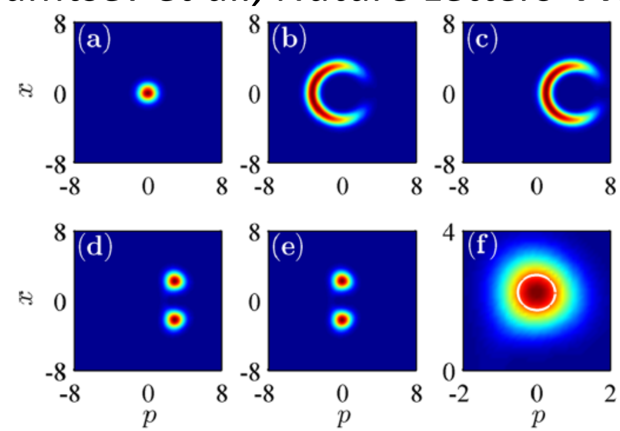
2. Cat-state source model



M. Brune et al., PRL **77**, 4887 (1996)



A. Ourjoumtsev et al., Nature Letters **448**, 784 (2007)



A. Negretti et al., PRL **99**, 223601 (2007)

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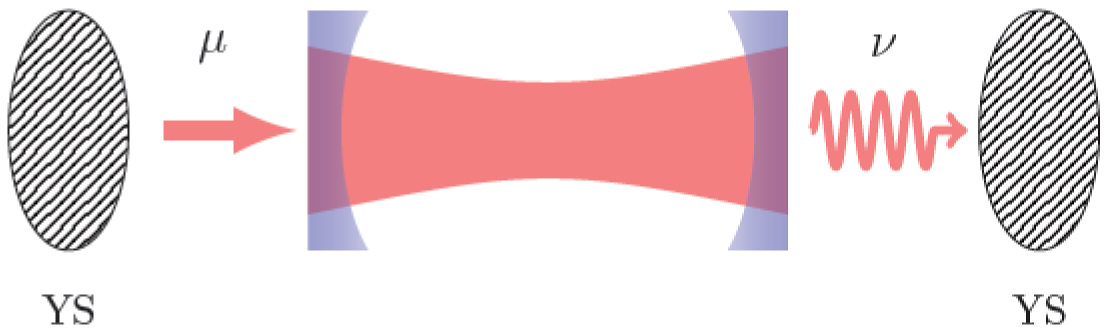


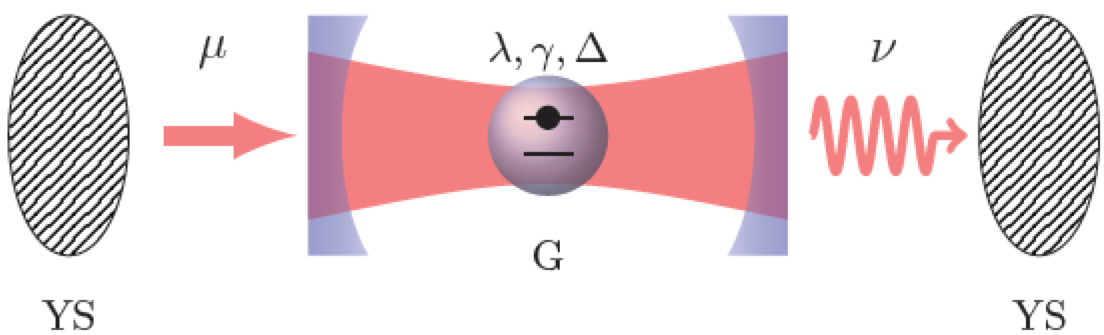
Analogy with Glauber coherent state generation:

Source (resonant oscillating dipole) + damping (Markovian photon loss) = coherent steady-state

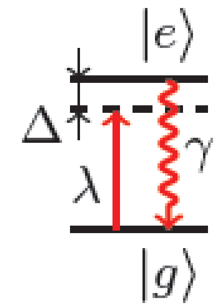
$$\partial_t \hat{\rho}_{ph} = \Lambda_{ph}[\hat{\rho}_{ph}] = -\imath\mu[\hat{a}_{YS} + \hat{a}^\dagger_{YS}, \hat{\rho}_{ph}] + 2\nu \hat{a}_{YS} \hat{\rho}_{ph} \hat{a}^\dagger_{YS} - \nu \left\{ \hat{a}^\dagger_{YS} \hat{a}_{YS}, \hat{\rho}_{ph} \right\}$$

$$\Lambda_{ph}[|\alpha_{st}\rangle_{YS}\langle\alpha_{st}|] = 0 \qquad \alpha_{st} = -\imath\mu/\nu$$

3. Single atom case: master equation



$$\hat{s}_+ = |e\rangle\langle g|$$
$$\hat{s}_- = \hat{s}_+^\dagger$$
$$\hat{s}_0 = \frac{[\hat{s}_+, \hat{s}_-]}{2}$$



Interaction with 2-level atom: $\hat{H}_{at} = \Delta \hat{s}_0$ $\quad \hat{H}_{int} = \lambda(\hat{a}_G \hat{s}_+ + \hat{a}_G^\dagger \hat{s}_-)$

$$\partial_t \hat{\rho}_{tot} = -\imath[\hat{H}_{at} + \hat{H}_{int}, \hat{\rho}_{tot}] + \Lambda_{ph}[\hat{\rho}_{tot}] + \Lambda_{at}[\hat{\rho}_{tot}]$$

$$\Lambda_{at}[\hat{\varrho}] = \gamma \hat{s}_- \hat{\varrho} \hat{s}_+ - \frac{\gamma}{2}\{\hat{s}_+ \hat{s}_-, \hat{\varrho}\}$$

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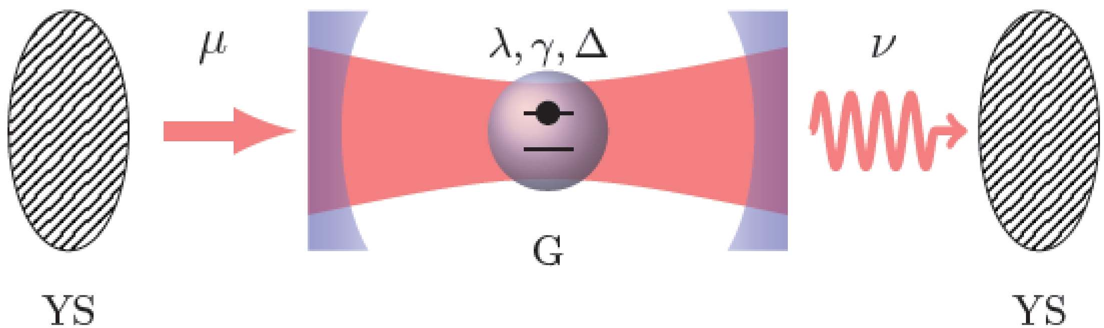

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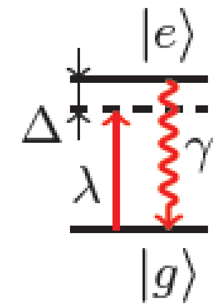

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Coherent feedback through *stimulated* excitations: $\hat{a}_G|\alpha\rangle_{YS} = \alpha|-\alpha\rangle_{YS}$

4. Single atom case: steady-state solution

Field has many photons:

$$|\alpha_{st}| \gg 1 \Rightarrow \mu \gg \nu \Rightarrow \langle\alpha_{st}| - \alpha_{st}\rangle \ll 1$$

Slow atomic evolution:

$$\mu, \nu \gg \Delta, \gamma, \lambda$$

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$$\Longrightarrow$$

Ansatz for the combined state:

$$\hat{\rho}_{tot} = \hat{\varrho}^{(+)} \otimes |\alpha_{st}\rangle_{YS}\langle\alpha_{st}| + \hat{\varrho}^{(-)} \otimes |-\alpha_{st}\rangle_{YS}\langle-\alpha_{st}| + \hat{R} \otimes |\alpha_{st}\rangle_{YS}\langle-\alpha_{st}| + \hat{R}^{\dagger} \otimes |-\alpha_{st}\rangle_{YS}\langle\alpha_{st}|$$

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$$\partial_t \hat{\varrho}^{(+)} = -\imath\Delta[\hat{s}_0, \hat{\varrho}^{(+)}] + \Lambda_{at}[\hat{\varrho}^{(+)}] + \imath\lambda\alpha(\hat{R}\hat{s}_+ + \hat{s}_+\hat{R}^{\dagger} + \hat{R}\hat{s}_- + \hat{s}_-\hat{R}^{\dagger})$$

$$\partial_t \hat{\varrho}^{(-)} = -\imath\Delta[\hat{s}_0, \hat{\varrho}^{(-)}] + \Lambda_{at}[\hat{\varrho}^{(-)}] - \imath\lambda\alpha(\hat{s}_-\hat{R} + \hat{R}^{\dagger}\hat{s}_- + \hat{s}_+\hat{R} + \hat{R}^{\dagger}\hat{s}_+)$$

$$\partial_t \hat{R} + \Gamma\hat{R} = -\imath\Delta[\hat{s}_0, \hat{R}] + \Lambda_{at}[\hat{R}] + \imath\lambda\alpha(\hat{s}_+\hat{\varrho}^{(-)} + \hat{s}_-\hat{\varrho}^{(-)} - \hat{\varrho}^{(+)}\hat{s}_+ - \hat{\varrho}^{(+)}\hat{s}_-)$$

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$$\partial_t \hat{R} + \Gamma\hat{R} = -\imath\Delta[\hat{s}_0, \hat{R}] + \Lambda_{at}[\hat{R}] + \imath\lambda\alpha(\hat{s}_+\hat{\varrho}^{(-)} + \hat{s}_-\hat{\varrho}^{(-)} - \hat{\varrho}^{(+)}\hat{s}_+ - \hat{\varrho}^{(+)}\hat{s}_-)$$

$$\Gamma = 4\mu^2/\nu \gg \mu \Rightarrow \partial_t \hat{R}\Big|_{t \gtrsim 1/\Gamma} = 0$$

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Steady-state:

$$\hat{\varrho}_{st}^{(+)} = \hat{\varrho}_{st}^{(-)} = \frac{\nu_3}{2(\gamma + 2\nu_3)}|e\rangle\langle e| + \frac{\nu_3 + \gamma}{2(\gamma + 2\nu_3)}|g\rangle\langle g|$$

$$\hat{R}_{st} = \frac{\nu_3}{2(\gamma + 2\nu_3)} \cdot \frac{\imath\lambda\alpha}{\Gamma + \gamma/2 + \imath\Delta}|e\rangle\langle g| - h.c.$$

$$\nu_3 = \frac{\lambda^2|\alpha_{st}|^2(2\Gamma + \gamma)}{(\Gamma + \gamma/2)^2 + \Delta^2}$$

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Classical expression: $S(\omega) \sim ReTr\left[\hat{s}_+(\omega)\hat{s}_-\hat{\rho}_{st}\right]$

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Modified expression (with initial conditions for new “Heisenberg operators”):

$$I : \hat{A}^{(+)}(t)\Big|_{t=0} = \hat{s}_+, \hat{A}^{(-)}(t)\Big|_{t=0} = 0,$$

$$II : \hat{A}^{(+)}(t)\Big|_{t=0} = 0, \hat{A}^{(-)}(t)\Big|_{t=0} = \hat{s}_+$$

$$S(\omega) \sim ReTr\left[\hat{A}^{(+)}(\omega)\Big|_I \hat{s}_-\hat{\varrho}_{st}^{(+)} + \hat{A}^{(-)}(\omega)\Big|_{II} \hat{s}_-\hat{\varrho}_{st}^{(-)} + \hat{A}^{(+)}(\omega)\Big|_{II} \hat{s}_-\hat{\varrho}_{st}^{(-)} + \hat{A}^{(-)}(\omega)\Big|_I \hat{s}_-\hat{\varrho}_{st}^{(+)}\right]$$

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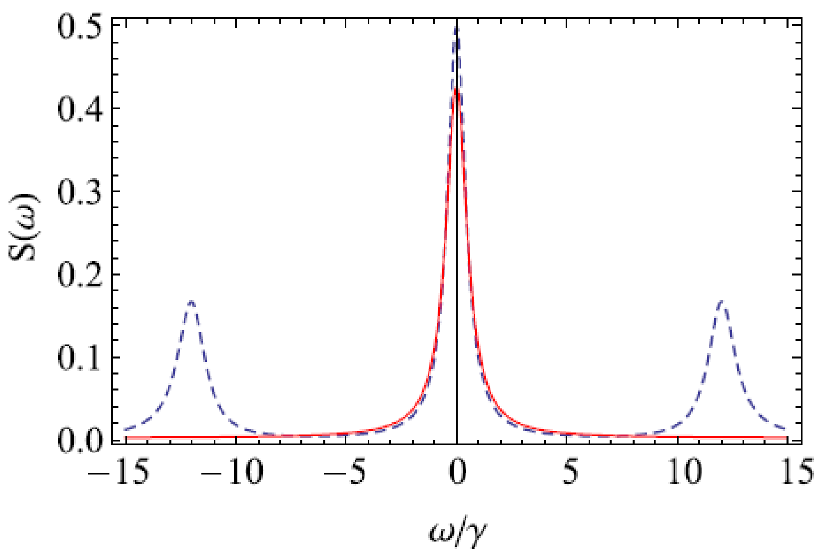
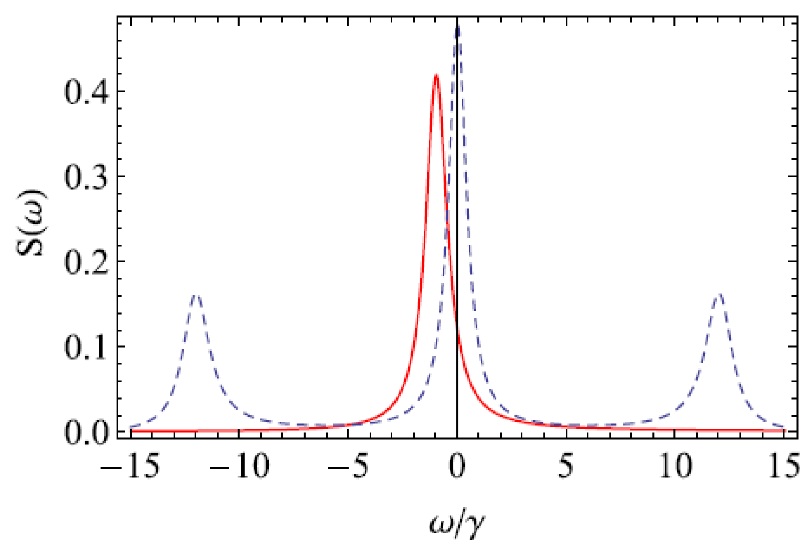
$\Delta = 0$



$\Delta = -\gamma$



6. Many atoms case: master equation & irreducible tensors

N 2-level atoms: $\hat{S}_m = \sum_{i=1}^{N} \hat{s}_m^{(i)};\;\; m = 0, \pm \quad \hat{H}_{at} = \Delta \hat{S}_0 \qquad \hat{H}_{int} = \lambda(\hat{a}_G \hat{S}_+ + \hat{a}_G^\dagger \hat{S}_-)$

Slow spontaneous emission: $\partial_t \hat{\rho}_{tot} = -\imath[\hat{H}_{at} + \hat{H}_{int}, \hat{\rho}_{tot}] + \Lambda_{ph}[\hat{\rho}_{tot}]$

Using the same ansatz: $\hat{\varrho} = \hat{\varrho}^{(+)} + \hat{\varrho}^{(-)} \quad \hat{r} = \hat{R} - \hat{R}^\dagger$

$$\partial_t \hat{\varrho} = -\imath\Delta[\hat{S}_0, \hat{\varrho}] + \imath\lambda\alpha_{st}[\hat{r}, \hat{S}_+ + \hat{S}_-]$$

$$\partial_t \hat{r} = -\imath\Delta[\hat{S}_0, \hat{r}] - \Gamma\hat{r} - \imath\lambda\alpha_{st}[\hat{\varrho}, \hat{S}_+ + \hat{S}_-]$$

In |jm>-basis: $\hat{\varrho}_{st} = \hat{1}/(N+1)$

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Angular momentum $j = N/2$ => convenient to use irreducible tensors:

$$\hat{O} = \sum_{\kappa=0}^{2j} \sum_{q=-\kappa}^{\kappa} O_{\kappa q} \hat{T}_{\kappa\, q}; \quad O_{\kappa q} = Tr\left(\hat{T}_{\kappa\, q}^\dagger \hat{O}\right) \qquad [\hat{S}_m, \hat{T}_{\kappa q}] = \sqrt{\kappa(\kappa+1)}\, C_{q,m,q+m}^{\kappa,1,\kappa} \hat{T}_{\kappa\, q+m}; \; m = 0, \pm 1$$

$$\partial_t \varrho_{\kappa\, q} = -\imath \Delta q \varrho_{\kappa\, q} + \left(C_{q+1,-1,q}^{\kappa,1,\kappa} F_{\kappa\, q+1} - C_{q-1,1,q}^{\kappa,1,\kappa} F_{\kappa\, q-1} \right)$$

$$F_{\kappa\, q} = \frac{2\lambda^2 \alpha_{st}^2 \kappa(\kappa+1)}{\Gamma + \imath \Delta q} \left(C_{q+1,-1,q}^{\kappa,1,\kappa} \varrho_{\kappa\, q+1} - C_{q-1,1,q}^{\kappa,1,\kappa} \varrho_{\kappa\, q-1} \right)$$

7. Many atoms case: 2nd-order correlation function

Heisenberg picture: $G(t) = Tr(\hat{S}_+ \hat{S}_+(t) \hat{S}_-(t) \hat{S}_- \varrho^{st})$

Schrödinger picture: $G(t) = Tr(\hat{S}_+ \hat{S}_- \hat{\varrho}(t)); \quad \hat{\varrho}(0) = \hat{S}_- \hat{\varrho}^{st} \hat{S}_+$

$$G(t) = \sum_{\kappa=0}^{2j} \sum_{q=-\kappa}^{\kappa} Tr(\hat{S}_+ \hat{S}_- \hat{T}_{\kappa q}) \varrho_{\kappa q}(t) = 2j(j+1) \sum_{\kappa=0}^{2j} \sum_{m=-j}^{j} \varrho_{\kappa\, 0}(t) \cdot (-1)^{j-m} \cdot (C^{j,1,j}_{m,-1,m-1})^2 \cdot C^{j,j,\kappa}_{m,-m,0}$$

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Approximation (smooth q-dependence of Clebsch-Gordan coefficients):

$$F_{\kappa\, q-1} = F_{\kappa\, q+1} \approx 2\lambda^2 \alpha_{st}^2 \kappa(\kappa+1) \varrho_{\kappa\, q} \cdot \left(\frac{C^{\kappa,1,\kappa}_{q,-1,q-1}}{\Gamma + \imath\Delta(q-1)} - \frac{C^{\kappa,1,\kappa}_{q,1,q+1}}{\Gamma + \imath\Delta(q+1)} \right)$$

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Approximate solution:

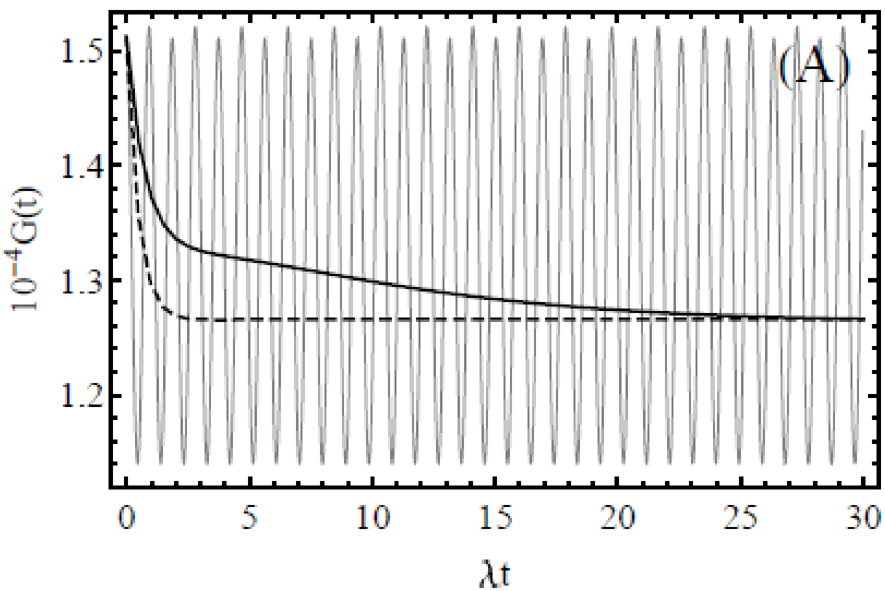
$$\varrho_{\kappa\, 0}(t) = \varrho_{\kappa\, 0}(0) \cdot exp(-A_\kappa t)$$

$$A_\kappa = \frac{4\Gamma\lambda^2 |\alpha_{st}|^2 \kappa(\kappa+1)}{\Gamma^2 + \Delta^2},$$

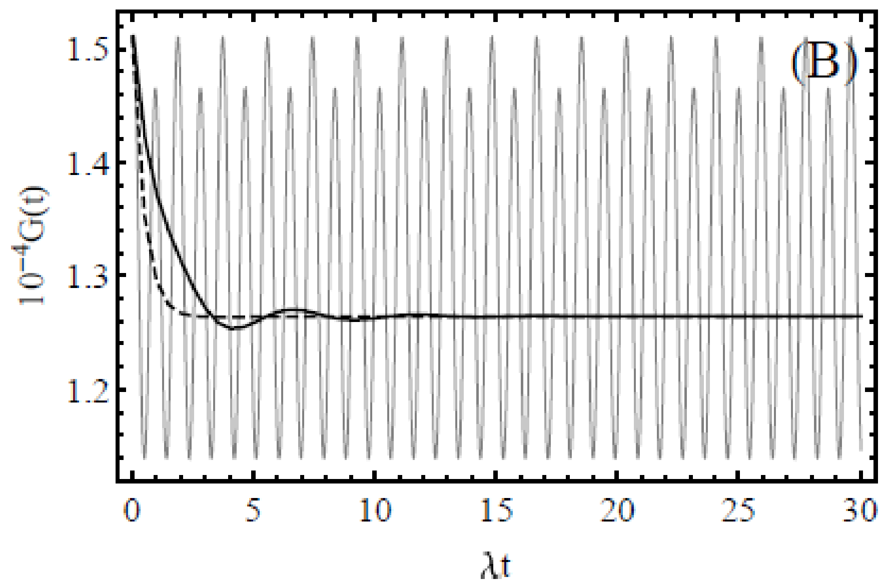
7. Many atoms case: 2^{nd}-order correlation function

$$\mu = 10\lambda, \nu = 3\lambda, \ N = 25$$

$\Delta = \lambda/10$



$\Delta = 2\lambda/3$



$$\lim_{t\to\infty} G(t) = \left(\frac{2j(j+1)}{3}\right)^2$$

Conclusions

- Steady-state interaction regime between atom(s) and cat-state field was studied.
- Spectrum of resonance fluorescence was calculated in the case of one atom.
- The (classical) correlations that build up between the atom and the field are responsible for suppressing sideband in the resonance fluorescence triplet.
- For atomic ensemble with many atoms, steady-state density matrix was obtained and 2^{nd}-order correlation function of atomic photoemissions was evaluated. The results differ drastically from the case of classical (coherent-state) field.

Thank you for your attention!