О возможности космологической гравиметрии с использование прецизионных атомных часов

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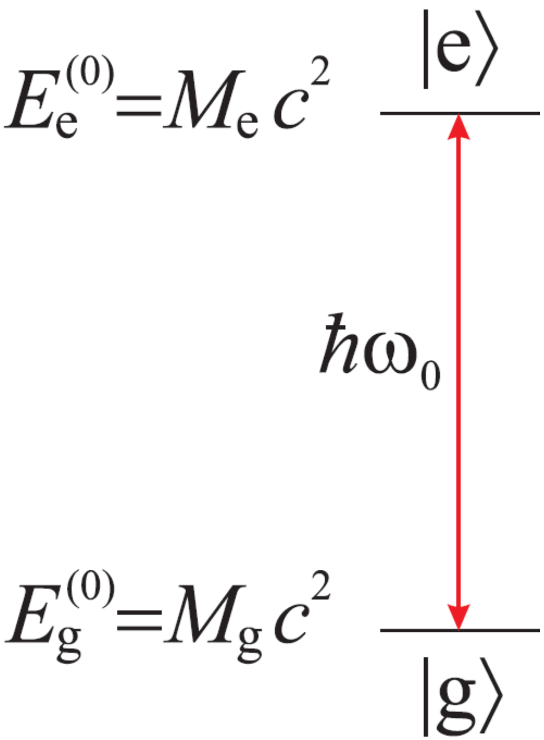
Previously we have developed the mass defect concept with respect to atomic clocks. Historically, considerations of the mass defect have been connected with nuclear physics, where the mass defect explains the huge energy emitted due to nuclear reactions. However, a quite unexpected result is that this effect has a direct relation to ultra-precise frequency standards, where it leads to shifts in the frequencies of reference atomic transitions.

Main idea of "quasi-classical" approach



Using Einstein's famous formula, $E=Mc^2$, which links the mass M and energy E of a particle (c is the speed of light), we can find the rest masses of our particle, M_g and M_e, for the states $|g\rangle$ and $|e\rangle$, respectively: $E_g = M_g c^2$ and $E_e = M_e c^2$.

The fact that $M_g \neq M_e$ is the essence of the so-called mass defect. In our case, the connection between M_g and M_e is as follows:

$$M_e c^2 = M_g c^2 + \hbar\omega_0 \quad \Rightarrow \quad M_e = M_g + \frac{\hbar\omega_0}{c^2}$$

Gravitational shift 1

We show how the mass defect allows us to formulate the simplest possible explanation of the gravitational redshift, even with a classical description of the gravitational field (as classical Newtonian potential U_G). Indeed, because the potential energy of a particle in a classical gravitational field is equal to the product MU_G (where $U_G < 0$), we can write the energy of *j-th* state as:

$$E_g = M_g c^2 + M_g U_G = M_g c^2 (1 + U_G / c^2)$$

$$E_e = M_e c^2 + M_e U_G = M_e c^2 (1 + U_G / c^2)$$

Then we find the frequency of the transition $|g\rangle \leftrightarrow |e\rangle$ in the gravitational field:

$$\omega = \frac{E_e(U_G) - E_g(U_G)}{\hbar} = \omega_0 \left(1 + \frac{U_G}{c^2}\right)$$

Gravitational shift 2

This expression coincides (to leading order) with a well known result based on general relativity theory:

$$\omega = \omega_0 \sqrt{1 + \frac{2U_G}{c^2}} \approx \omega_0 \left(1 + \frac{U_G}{c^2}\right)$$

Thus, the combination of special relativity ($E = Mc^2$) , quantum mechanics (definition of the frequency of atomic transitions) and Newtonian gravity leads to a possible explanation of the gravitational shift, which describes different experiments with atomic clocks in a spatially nonuniform gravitational field $U_G(\mathbf{r})$. We emphasize that the gravitational shift was derived without including the time dilation, which is taken as a basis of Einstein's theory of relativity. Therefore, the developed mass defect approach will be reffered to as "quasi-classical" explanation of gravitational shift.

Quasi-classical consideration of chronometric gravimetry

Lets start with "quasi-classical" expression for the transition frequency in the presence of Newtonian gravitational potential:

$$\omega(\mathbf{r}) = \omega_0 \left(1 + \frac{\varphi(\mathbf{r})}{c^2}\right)$$

We assume that a clock frequency is locked to this frequency. The next step consists in the formulation of a nonlocal chronometric principle: **The measured ratio of frequencies of the same clock transition in two different locations measured with respect to the same frequency standard is independent on the point where this standard is located**:

$$\frac{\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = 1 - \frac{1}{1 + \varphi(\mathbf{r}_1)/c^2} \frac{\varphi(\mathbf{r}_1) - \varphi(\mathbf{r}_2)}{c^2} \equiv fixed$$

Absoluteness of gravitational potential

Usually assumed that the gravitational potential is determined with accuracy to arbitrary constant *C*: The substitution

$$\varphi(\mathbf{r}) \to \varphi(\mathbf{r}) + C$$

does not change the force $\mathbf{F}_{\text{grav}} = -M\nabla\varphi(\mathbf{r})$

In such a case the ratio $\omega(\mathbf{r}_1)/\omega(\mathbf{r}_2)$

will be uncertain, but we can measure it!

If the nonlocal chronometric principle is true (confirmed by experiments), then transformation $\varphi(\mathbf{r}) \to \varphi(\mathbf{r}) + C$ *is not possible, i.e.* $\varphi(\mathbf{r})$ *is some uniquely defined function for the Universe.*

Chronometric gravimetry 3

Let us consider now the expression:

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)}=\frac{1}{1+\varphi(\mathbf{r}_1)/c^2}\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$

where

$$\varphi(\mathbf{r})=\varphi_{\mathrm{loc}}(\mathbf{r})+\Phi_{\mathrm{CP}}(\mathbf{r})\,,$$

$$\varphi_{\mathrm{loc}}(\mathbf{r})=\varphi_{\mathrm{E}}(\mathbf{r})+\varphi_{\mathrm{S}}(\mathbf{r})+\varphi_{\mathrm{M}}(\mathbf{r})+\ldots$$

$$|\varphi_{\mathrm{E}}/c^2|\approx 0.7\times 10^{-9} \qquad |\varphi_{\mathrm{S}}/c^2|\approx 10^{-8}$$

Rough estimates leads to $|\Phi_{\mathrm{CP}}| \gg |\varphi_{\mathrm{loc}}(\mathbf{r})|$

$$|\Phi_{\mathrm{CP}}/c^2| > v_S^2/c^2 \sim 10^{-6} \qquad |\Phi_{\mathrm{CP}}/c^2| \gg 10^{-6}$$

Chronometric gravimetry 4

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} \approx \frac{1}{1+\Phi_{\mathrm{CP}}/c^2}\,\frac{\varphi_{\mathrm{loc}}(\mathbf{r}_1)-\varphi_{\mathrm{loc}}(\mathbf{r}_2)}{c^2}$$

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1+\alpha)\,\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$

$$\alpha = -\frac{\Phi_{\mathrm{CP}}/c^2}{1+\Phi_{\mathrm{CP}}/c^2} \quad \Leftrightarrow \quad \Phi_{\mathrm{CP}}/c^2 = -\frac{\alpha}{1+\alpha}$$

$$|\Phi_{\mathrm{CP}}/c^2| \ll 1$$

$$\alpha \approx -\Phi_{\mathrm{CP}}/c^2 \; > 0$$

What about general relativity?

It can be shown that the orthodox GR sais that α=0. Note also that it is nontrivial result, which requires rigorous derivation.

$$ds^2 = g_{jk}(\vec{x})dx^j dx^k \quad (j,k = 0,1,2,3)$$

$$\frac{\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = \sqrt{\frac{g_{00}(\mathbf{r}_2)}{g_{00}(\mathbf{r}_1)}} = \sqrt{1 - \frac{g_{00}(\mathbf{r}_1) - g_{00}(\mathbf{r}_2)}{g_{00}(\mathbf{r}_1)}}$$

$$\frac{\omega(\mathbf{r}_1) - \omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = 1 - \frac{\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = 1 - \sqrt{\frac{g_{00}(\mathbf{r}_2)}{g_{00}(\mathbf{r}_1)}} \approx (1+\alpha)\frac{\varphi(\mathbf{r}_1) - \varphi(\mathbf{r}_2)}{c^2}$$

$$\begin{aligned} ds^2 &= -c^2\left(1 + \frac{2\varphi}{c^2} + \frac{2\varphi^2}{c^4} + O(c^{-6})\right)dt^2 \\ &\quad + \left(1 - \frac{2\varphi}{c^2} + O(c^{-4})\right)(d\mathbf{r})^2. \end{aligned}$$

α=0

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1+\alpha)\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$

But there exist generalized theories, with nonzero α. Moreover it was assumed that α may depend on the type of transition. These facts were already tested in many experiments, including those with atomic clock on rockets, airplanes and satellites. The main results: the difference of the two α is zero with accuracy about 10^{-7}, but what about α itself, no certain answer, just upper limit $\alpha < 2\times10^{-4}$.

C. M. Will, Living Rev. Relativity 17, 4 (2014).

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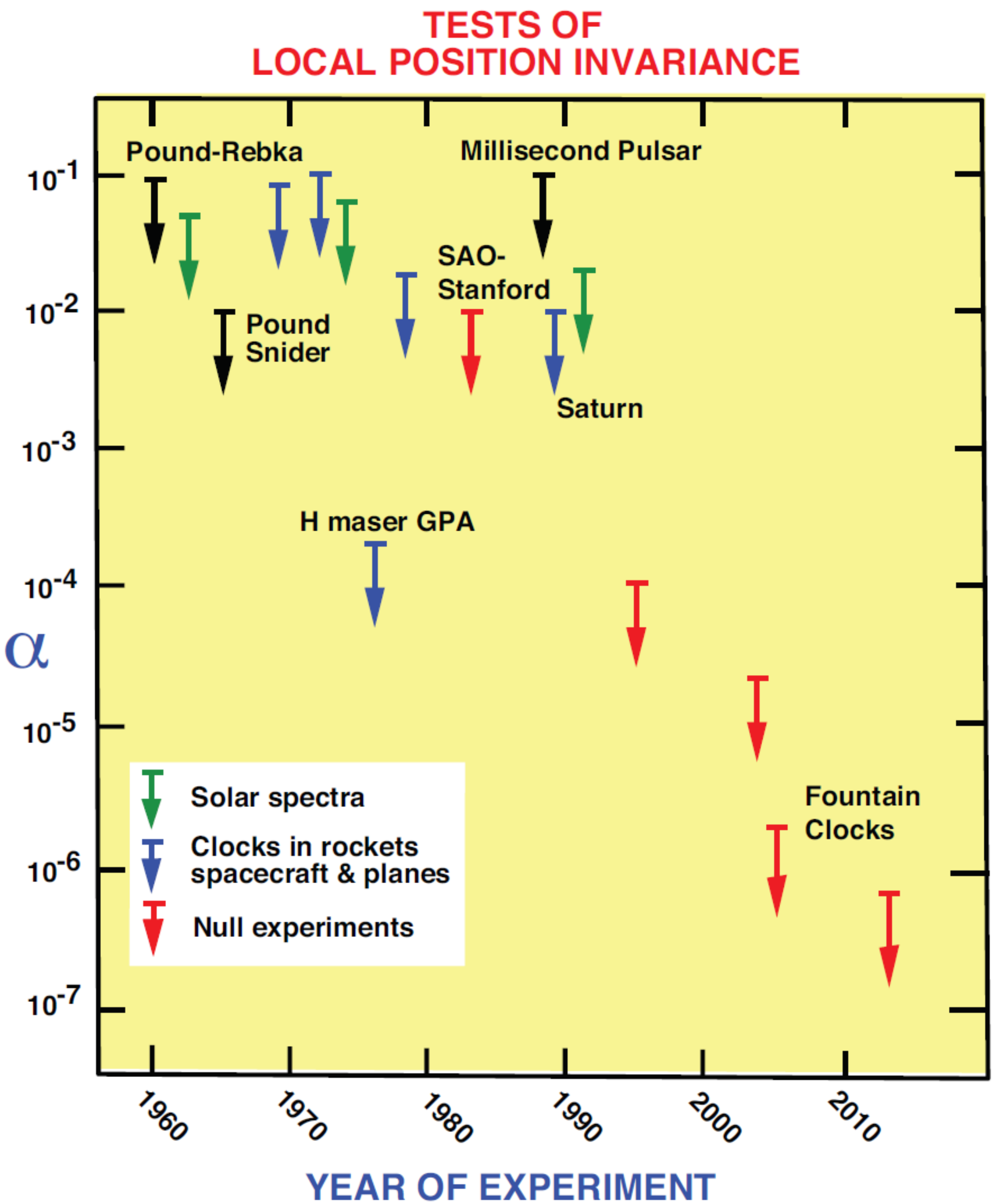

$$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$$

In null redshift experiments it is assumed that nonzero $\alpha \neq 0$ depends on the type of atomic clocks. Then, comparing the behavior of two different clocks based on different atomic transitions (with frequencies $\omega^{(1)}$ and $\omega^{(2)}$), one can measure with high accuracy the difference between the corresponding coefficients, $(\alpha_1 - \alpha_2)$:

$$\frac{\Delta\omega^{(1)}}{\omega} - \frac{\Delta\omega^{(2)}}{\omega} = (\alpha_1 - \alpha_2)\frac{\Delta\varphi}{c^2}$$

The main idea of null redshift experiments: if the experiments show that $(\alpha_1 - \alpha_2) \to 0$, then $\alpha_{1,2} \to 0$ that is, the principle of local invariance is not violated. However, as shown above, the presence of $\alpha \neq 0$ can be associated with cosmological gravitation, where α is universal and does not depend on the type of atomic clock.

Experiment of H. Katori, 2015

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Geopotential measurements with synchronously linked optical lattice clocks

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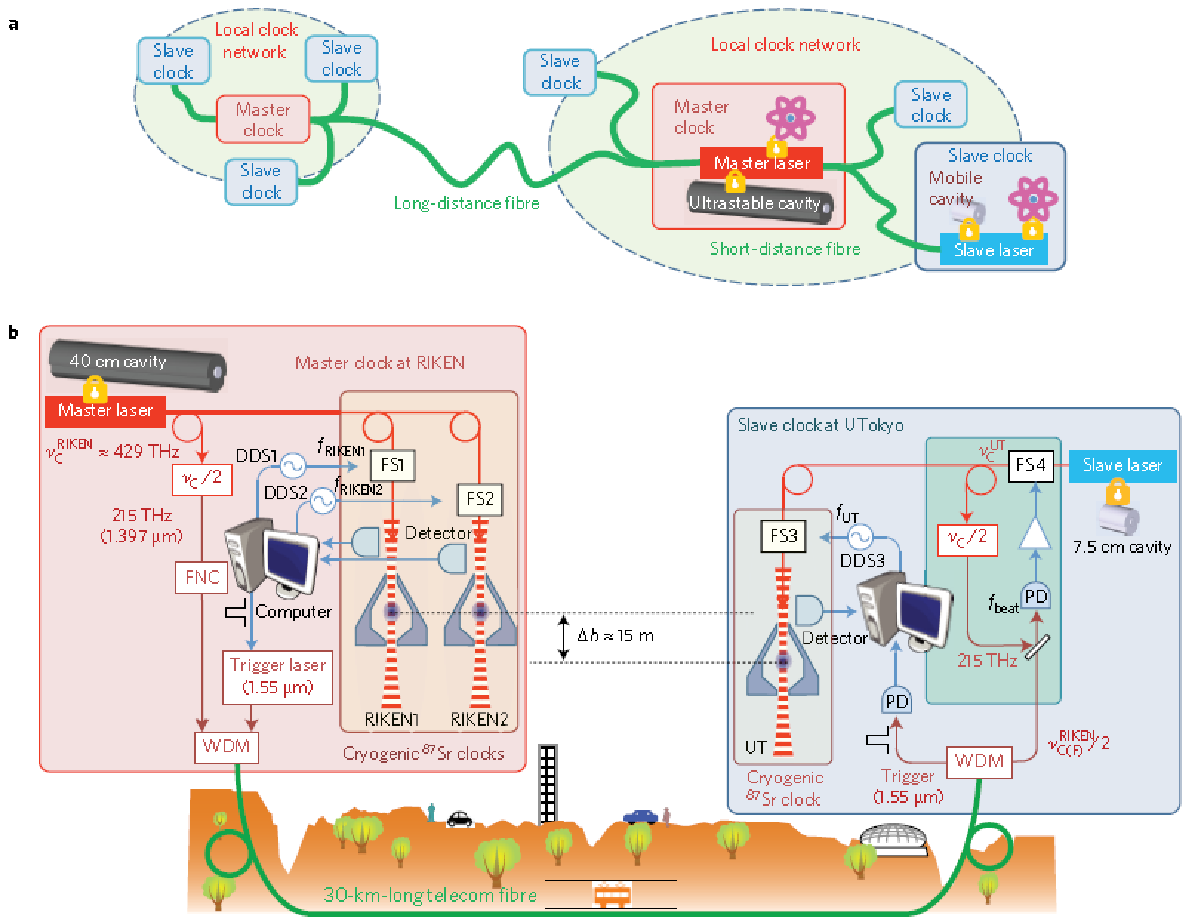
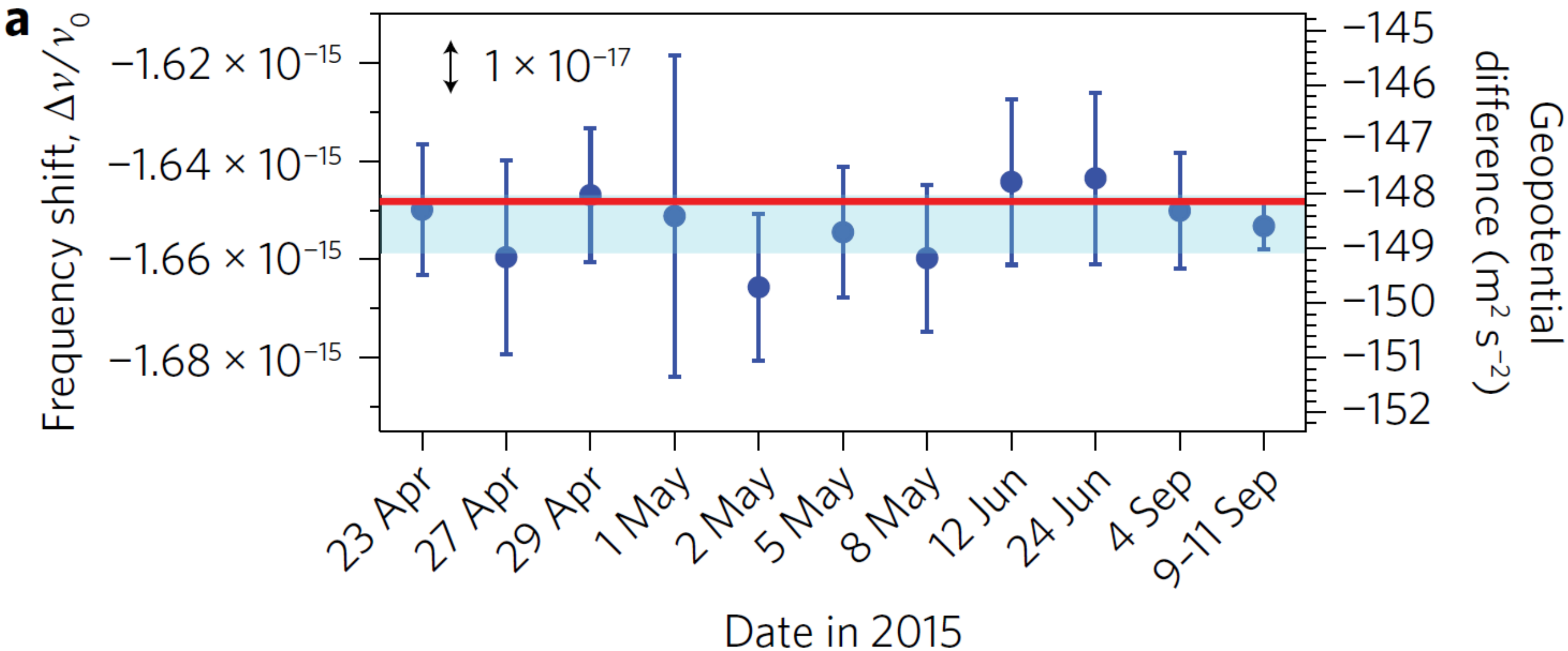


Frequency difference vs potential difference

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1+\alpha)\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$



Our comment: we see a systematic discrepancy between chronometric (blue) and geodetic (red) data. It can be attributed to nonzero $\alpha > 10^{-3}$! ($\alpha = 2.5\times10^{-3}$ for blue midline). But the chronometric data scattering is large.

Our proposal

- Repeat an experiment like Katori's one, but with two identical atomic clocks at different height, but at the same geographical point in order to reduce the time-dependent influences of Sun and Moon.
- Experimentally determine α and Φ_{CP}
- or, at least, establish new more stringent upper limits for them.

Estimations

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1+\alpha)\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$

$$\frac{\Delta\omega}{\omega} - \frac{\Delta\varphi}{c^2} = \alpha\frac{\Delta\varphi}{c^2}$$

$$\Delta\varphi = \varphi(z_1) - \varphi(z_2) = -\int_{z_1}^{z_1+h} g(z)dz$$

$$\Delta\varphi \approx -hg(z_1)$$

If the relative uncertainty of the atomic clocks is 10^{-18} and measurement accuracy of *h* is 1 cm:

$\alpha>10^{-3}$ for h=10m; **$\alpha>10^{-4}$** for h=100m; **$\alpha>10^{-5}$** for h=1000m

If the relative uncertainty of the atomic clocks is 10^{-19} and measurement accuracy of *h* is 1 mm:

$\alpha>10^{-4}$ for h=10m; **$\alpha>10^{-5}$** for h=100m; **$\alpha>10^{-6}$** for h=1000m

Resume

$$\frac{\omega(\mathbf{r}_1)-\omega(\mathbf{r}_2)}{\omega(\mathbf{r}_1)} = (1+\alpha)\frac{\varphi(\mathbf{r}_1)-\varphi(\mathbf{r}_2)}{c^2}$$

1. If experiments will show universal $\alpha > 10^{-6}$, then it will be Cosmological Gravimetry.

2. If experiments will show $\alpha = 0$ (at least $|\alpha| << 10^{-6}$), then it will be one of best tests of General Relativity.

3. If experiments will show negative value of universal α ($\alpha < 0$ and $|\alpha| > 10^{-6}$), then it will be absolutely non-understandable.

Thank you very much for your attention!