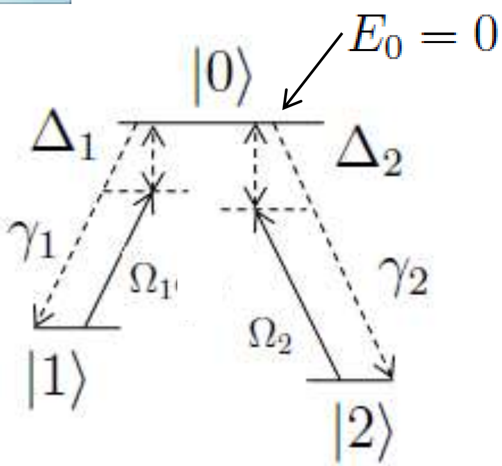
Спектроскопия Λ-атома с использованием обратной связи

В. А. Томилин, Л. В. Ильичёв
ИАиЭ СО РАН
НГУ

1. Трехуровневая система.



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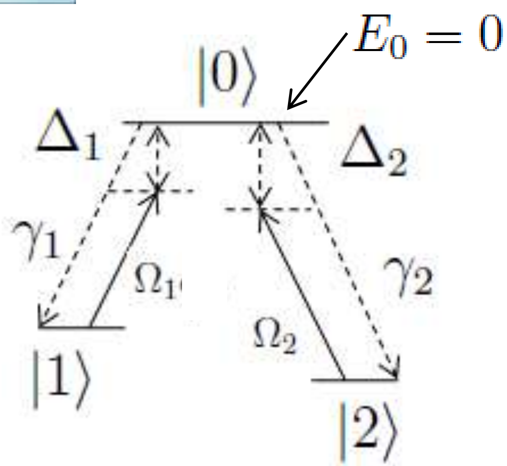

$$\hat{H}_{tot} = \hat{H}_\Lambda + \hat{V}_1 + \hat{V}_2$$

$$\hat{H}_\Lambda = \sum_{i=1,2} \Delta_i |i\rangle\langle i| \qquad \hat{V}_i = \Omega_i |0\rangle\langle i| + h.c$$

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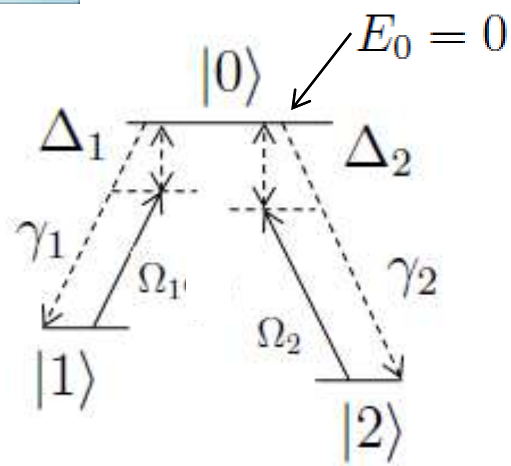

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Когерентное пленение населенностей (КПН):

$$\Delta_1 = \Delta_2 \qquad |\Psi_{dark}\rangle = \frac{\Omega_2|1\rangle - \Omega_1|2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}} \qquad |\Psi_{bright}\rangle = \frac{\Omega_1|1\rangle + \Omega_2|2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}$$

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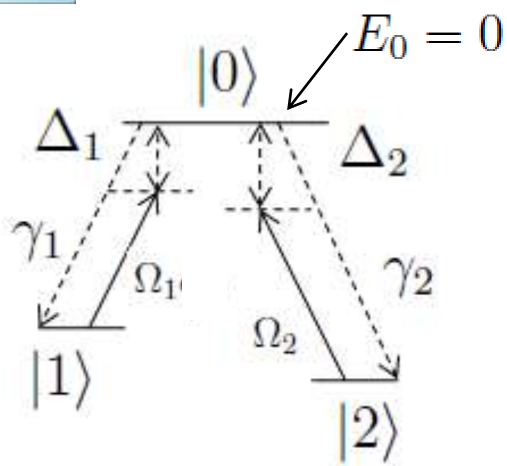

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Учет спонтанного испускания:

$$\hat{L}_i = \sqrt{\gamma_i}|i\rangle\langle 0|; i = 1, 2 \qquad \partial_t \hat{\rho} = -\imath[\hat{H}_{tot}, \hat{\rho}] + \sum_{i=1,2} \left(\hat{L}_i \hat{\rho} \hat{L}_i^\dagger - \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}\} \right)$$

2. «Альтернативное распутывание» операции фотодетектирования.

$$\{\hat{L}_1, \hat{L}_2\} \to \{\hat{L}_+, \hat{L}_-\}$$

$$\hat{L}_+ = \alpha \hat{L}_1 + \beta \hat{L}_2,$$

$$\alpha \in \mathbb{R}$$

$$\hat{L}_- = -\beta^* \hat{L}_1 + \alpha^* \hat{L}_2,$$

$$|\alpha|^2 + |\beta|^2 = 1.$$

$$\sum_{i=1,2} \left(\hat{L}_i \hat{\rho} \hat{L}_i^\dagger - \frac{1}{2} \{\hat{L}_i^\dagger \hat{L}_i, \hat{\rho}\} \right) = \sum_{\sigma=+,-} \left(\hat{L}_\sigma \hat{\rho} \hat{L}_\sigma^\dagger - \frac{1}{2} \{\hat{L}_\sigma^\dagger \hat{L}_\sigma, \hat{\rho}\} \right)$$

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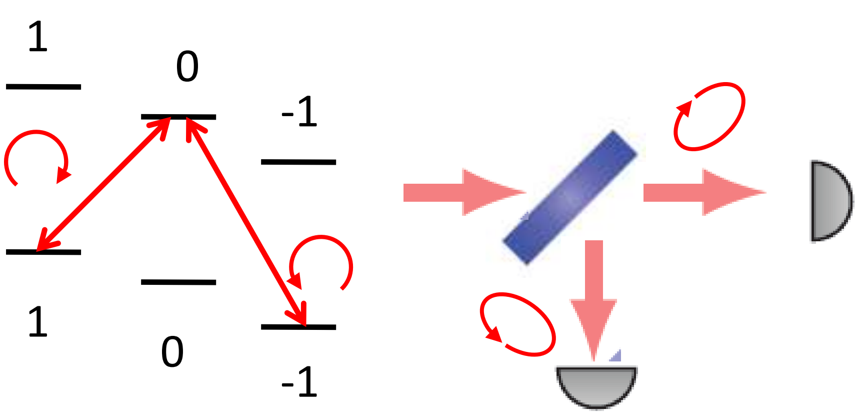
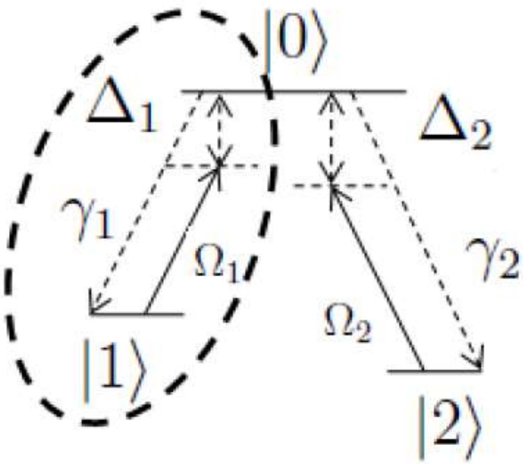
Реализация в одномерном случае:



3. Схема действия обратной связи. Управляющие уравнения.



Событие типа "+" ($\hat{\rho} \to \dfrac{\hat{L}_{+}\hat{\rho}\hat{L}_{+}^{\dagger}}{Tr\left[\hat{L}_{+}\hat{\rho}\hat{L}_{+}^{\dagger}\right]}$): $Arg\Omega_1 \to 0$

Событие типа "-" ($\hat{\rho} \to \dfrac{\hat{L}_{-}\hat{\rho}\hat{L}_{-}^{\dagger}}{Tr\left[\hat{L}_{-}\hat{\rho}\hat{L}_{-}^{\dagger}\right]}$): $Arg\Omega_1 \to \pi$

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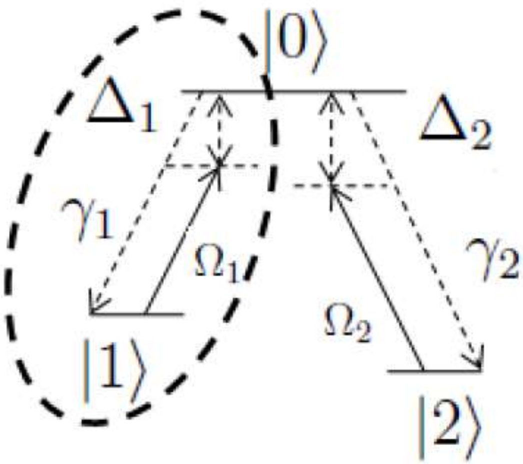

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$$\hat{\rho} \to \{\hat{\rho}^{(+)}, \hat{\rho}^{(-)}\} \qquad \hat{\rho}_{tot} = \sum_\sigma \hat{\rho}^{(\sigma)} \qquad \hat{H}_{tot}^{(\pm)} = \hat{H}_\Lambda \pm \hat{V}_1 + \hat{V}_2$$

$$\partial_t \hat{\rho}^{(+)} = -\imath[\hat{H}_{tot}^{(+)}, \hat{\rho}^{(+)}] + \hat{L}_+(\hat{\rho}^{(+)} + \hat{\rho}^{(-)})\hat{L}_+^\dagger - \frac{1}{2}\sum_{\sigma=+,-}\{\hat{L}_\sigma^\dagger \hat{L}_\sigma, \hat{\rho}^{(+)}\},$$

$$\partial_t \hat{\rho}^{(-)} = -\imath[\hat{H}_{tot}^{(-)}, \hat{\rho}^{(-)}] + \hat{L}_-(\hat{\rho}^{(+)} + \hat{\rho}^{(-)})\hat{L}_-^\dagger - \frac{1}{2}\sum_{\sigma=+,-}\{\hat{L}_\sigma^\dagger \hat{L}_\sigma, \hat{\rho}^{(-)}\}.$$

4. Анализ стационарного решения.

Случай точного резонанса: $\Delta_1 \to \Delta_2$

Два типа темных и светлых состояний:

$$|\Psi^{(\pm)}_{dark}\rangle = \frac{\Omega_2|1\rangle \mp \Omega_1|2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}},$$

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Структура решения: $\hat{\rho}^{(\sigma)}_{st} = p^{(st)}_{\sigma}|\Psi^{(\sigma)}_{dark}\rangle\langle\Psi^{(\sigma)}_{dark}|$

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Состояния после фоторегистрации: $\dfrac{\hat{L}_\sigma\hat{\rho}\hat{L}^\dagger_\sigma}{Tr\hat{L}_\sigma\hat{\rho}\hat{L}^\dagger_\sigma} = |\Psi^{(\sigma)}_{post}\rangle\langle\Psi^{(\sigma)}_{post}|$

$$\frac{\alpha}{\beta} = \frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}} : |\Psi^{(+)}_{post}\rangle = |\Psi^{(+)}_{bright}\rangle \Rightarrow \hat{\rho}^{(+)} = 0, \hat{\rho}^{(-)} = |\Psi^{(-)}_{dark}\rangle\langle\Psi^{(-)}_{dark}|$$

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5. Работа полей в стационарном случае.

Без о.с.: $A_i^{(nf)} \sim -\imath Tr\left([|i\rangle\langle i|, \hat{D}_i]E_i\hat{\rho}_{st}\right) \sim Re(\imath\Omega_i\rho_{0i})$

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С о.с.:

$$A_1 \sim Re(\imath\Omega_1(\rho_{01}^{(+)} - \rho_{01}^{(-)})),$$

$$A_2 \sim Re(\imath\Omega_2(\rho_{02}^{(+)} + \rho_{02}^{(-)})),$$

$$A = A_1/\gamma_1 = A_2/\gamma_2$$

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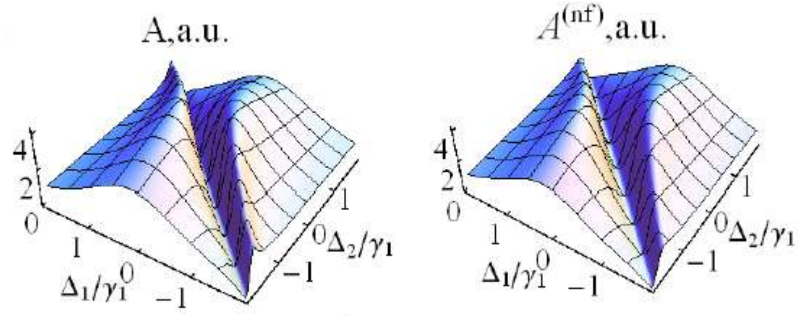
$$\Omega_2 = 0.67\gamma_1$$



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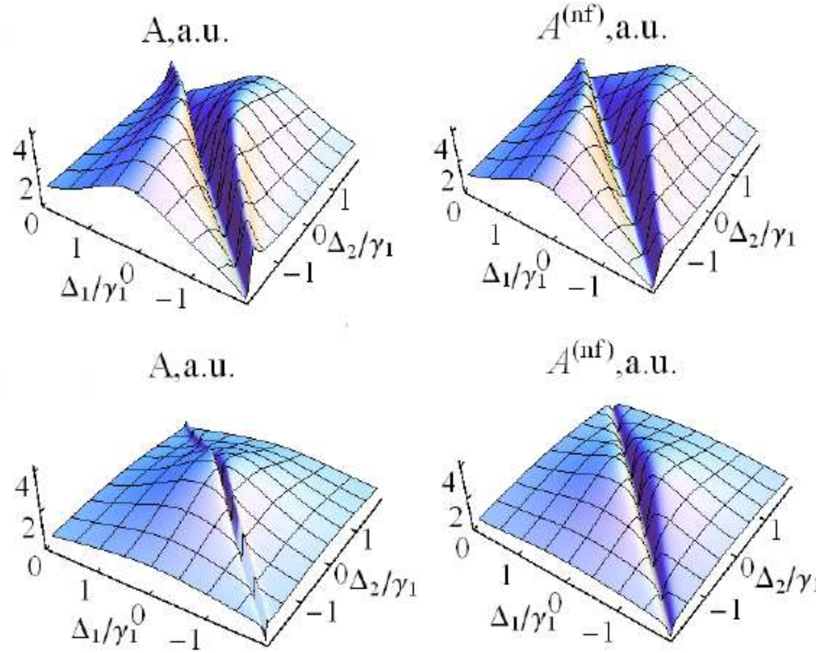

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Особая точка:

$$\frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}} = 1$$

$$\alpha = \beta = 1/\sqrt{2}$$

$$\Downarrow$$

$$|\Psi_{post}^{(\pm)}\rangle = |\Psi_{bright}^{(\pm)}\rangle$$

$\alpha = \beta = 1/\sqrt{2}, \gamma_1 = \gamma_2$

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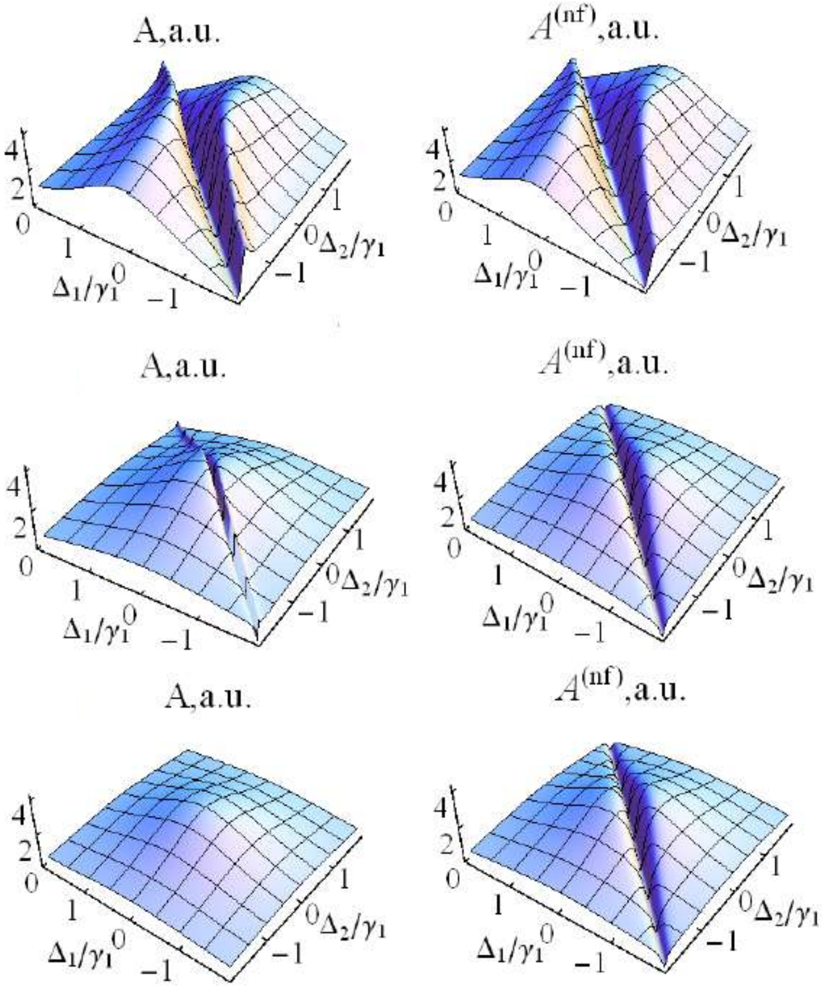

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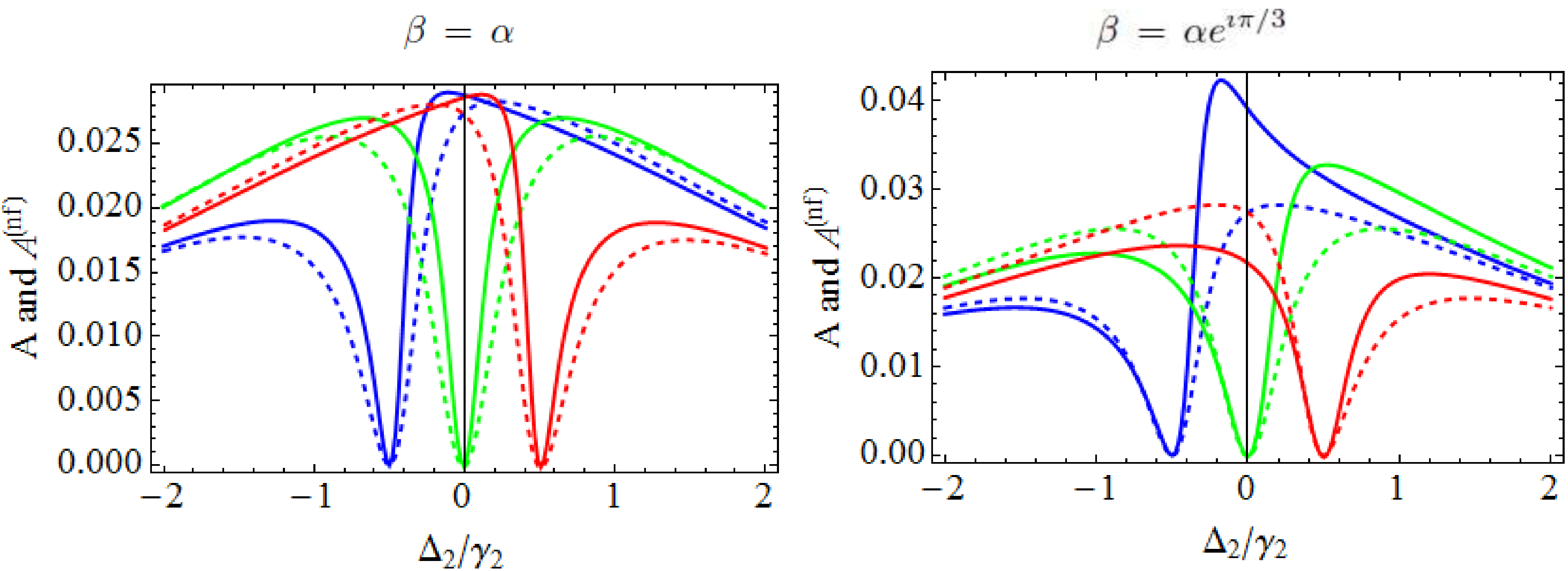

$\Delta_1 = 0$

$\Omega_1 = 1/5\gamma_1, \Omega_2 = 1/2\gamma_1, \gamma_2 = \gamma_1$

$\Delta_1 = -\gamma_1/2$

$\Delta_1 = \gamma_1/2$

С о.с.

Без о.с.

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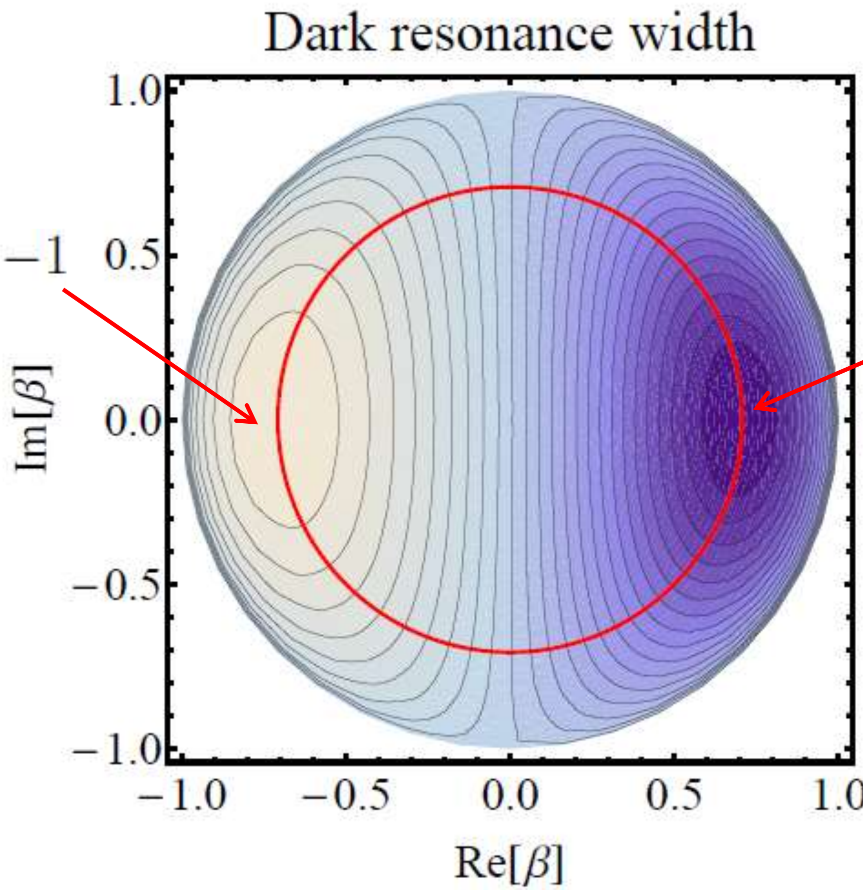

$\alpha/\beta = -1$

$\Downarrow$

$min|\langle\Psi^{(\sigma)}_{post}|\Psi^{(\sigma)}_{bright}\rangle|^2$

$\alpha/\beta = 1$

$\Downarrow$

$max|\langle\Psi^{(\sigma)}_{post}|\Psi^{(\sigma)}_{bright}\rangle|^2$

$$\Omega_1 = \gamma_1, \Omega_2 = 4\gamma_1, \gamma_2 = 2\gamma_1, \Delta_1 = 0$$

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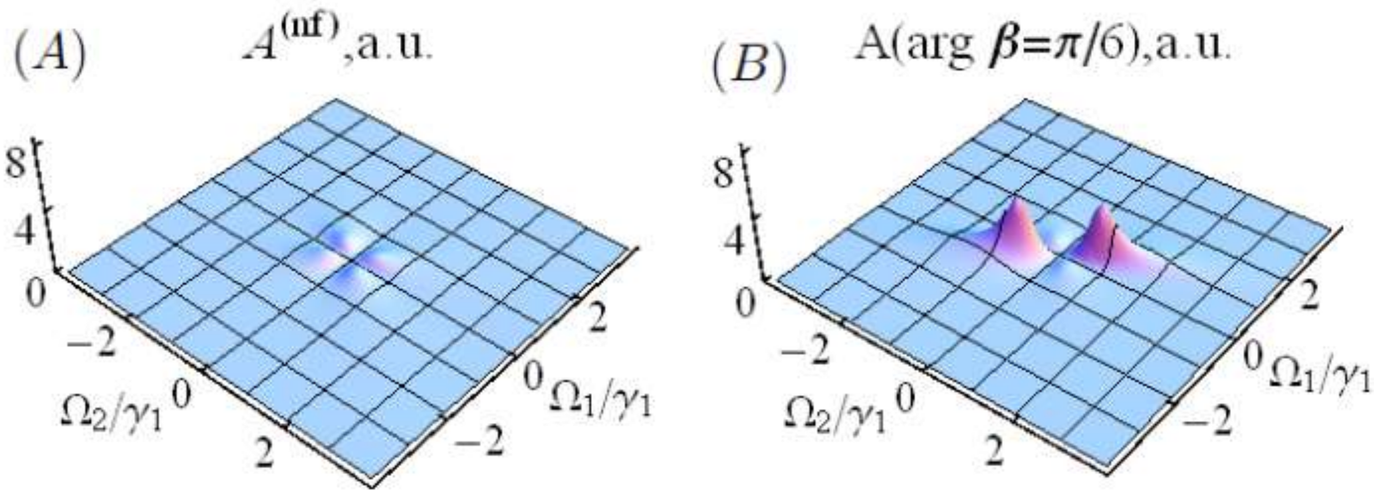

Min. width:

$$\frac{\alpha}{\beta} = \frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}}$$

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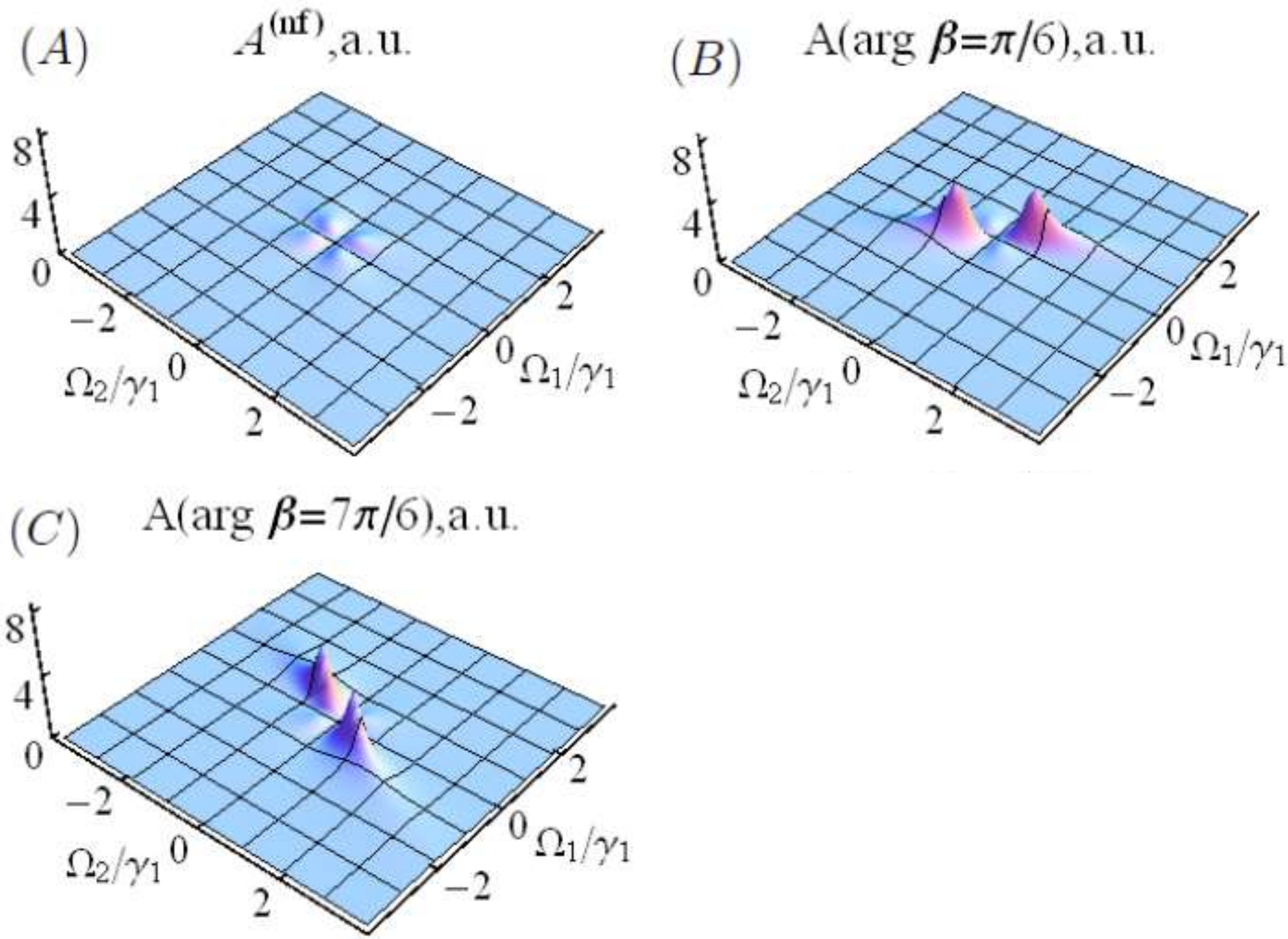

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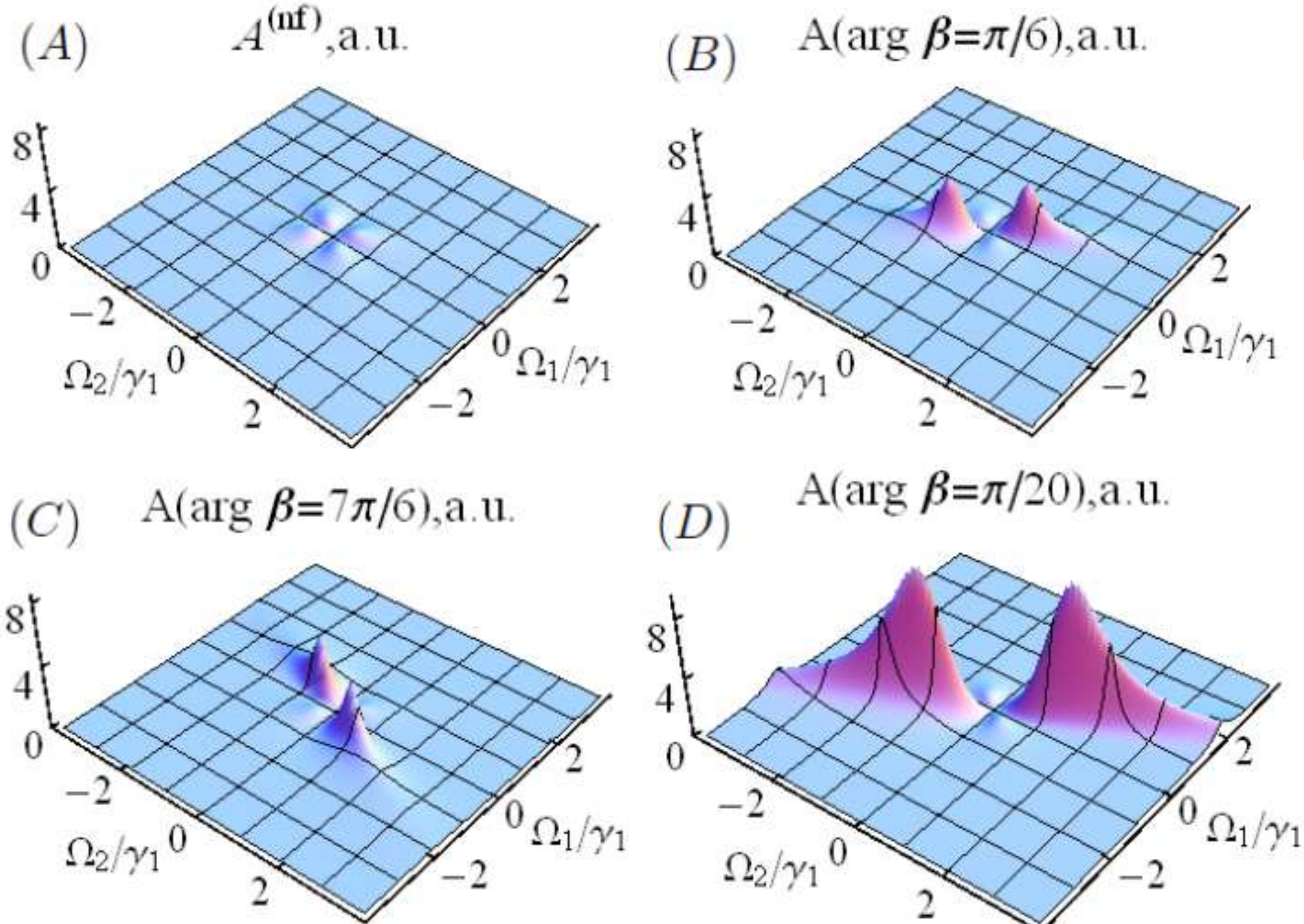

6. Заключение.

1. Исследована схема управления трехуровневой системой в Λ-конфигурации при помощи обратной связи, инициируемой нетривиальным распутыванием операции фотодетектирования.

2. Продемонстрирована возможность управления формой и шириной темных резонансов в трехуровневой Λ-схеме.

3. В зависимости работ полей от частот Раби обнаружены необычные узкие структуры резонансного типа.

Спасибо за внимание!

$$\frac{\alpha}{\beta} = -\frac{\Omega_2\sqrt{\gamma_2}}{\Omega_1\sqrt{\gamma_1}} : |\Psi^{(+)}_{post}\rangle = |\Psi^{(+)}_{dark}\rangle, \qquad |\Psi^{(-)}_{post}\rangle \propto \Omega_1\gamma_1|1\rangle + \Omega_2\gamma_2|2\rangle$$

$$\frac{\alpha}{\beta} = -\frac{\Omega_1\sqrt{\gamma_1}}{\Omega_2\sqrt{\gamma_2}} : |\Psi^{(-)}_{post}\rangle = |\Psi^{(-)}_{dark}\rangle.$$

$$\rho_{00}^{(+)} = \frac{4}{D} \cdot (\Delta_1 - \Delta_2)^2 \Omega_1^2 \Omega_2^2 (\alpha^2 \gamma_1 + |\beta|^2 \gamma_2),$$

$$\rho_{11}^{(+)} = \frac{\Omega_2^2}{D} [4|\beta|^2 \gamma_2 \Omega_1^2 (\Omega_1^2 + \Omega_2^2) + 4\alpha^2 \gamma_1 (\Delta_1 - \Delta_2)^2 (\Delta_1^2 + \Omega_1^2) +$$

$$4\alpha \Omega_1 \Omega_2 \sqrt{\gamma_1 \gamma_2} \left(Im\beta \cdot (\gamma_1 + \gamma_2)(\Delta_1 - \Delta_2) - 2Re\beta \cdot \left(\Omega_1^2 + \Omega_2^2 + \Delta_1(\Delta_2 - \Delta_1) \right) \right) +$$

$$4\alpha^2 \gamma_1 \Omega_2^2 (\Omega_1^2 + \Omega_2^2 + 2\Delta_1(\Delta_2 - \Delta_1)) + \alpha^2 \gamma_1 (\gamma_1 + \gamma_2)^2 (\Delta_1 - \Delta_2)^2],$$

$$\rho_{01}^{(+)} = \frac{2}{D} (\Delta_1 - \Delta_2) \Omega_1 \Omega_2^2 [\imath \alpha^2 \gamma_1 (\gamma_1 + \gamma_2)(\Delta_1 - \Delta_2) - 2(\alpha^2 \gamma_1 \Omega_2^2 + |\beta|^2 \gamma_2 \Omega_1^2) +$$

$$2\alpha^2 \gamma_1 \Delta_1 (\Delta_1 - \Delta_2) + 4\alpha Re\beta \sqrt{\gamma_1 \gamma_2} \Omega_1 \Omega_2],$$

$$\rho_{12}^{(+)} = -\frac{2\Omega_1 \Omega_2}{D} [- 2\alpha \Omega_1 \Omega_2 \sqrt{\gamma_1 \gamma_2} \left(2Re\beta(\Omega_1^2 + \Omega_2^2) + \imath\beta(\gamma_1 + \gamma_2)(\Delta_1 - \Delta_2) \right) +$$

$$2\alpha^2 \Omega_2^2 \gamma_1 \left(\Omega_1^2 + \Omega_2^2 + \Delta_1(\Delta_2 - \Delta_1) \right) + 2|\beta|^2 \Omega_1^2 \gamma_2 \left(\Omega_1^2 + \Omega_2^2 + \Delta_2(\Delta_1 - \Delta_2) \right) +$$

$$\imath(\gamma_1 + \gamma_2)(\Delta_1 - \Delta_2)(\alpha^2 \Omega_2^2 \gamma_1 + |\beta|^2 \Omega_1^2 \gamma_2)],$$

$$D \cdot \rho_{00,11,22,02}^{(-)} = (D \cdot \rho_{00,11,22,02}^{(+)}) \Big|_{\alpha^2 \leftrightarrow |\beta|^2} ; D \cdot \rho_{22,02}^{(\sigma)} = (D \cdot \rho_{11,01}^{(\sigma)}) \Big|_{1 \leftrightarrow 2; \alpha^2 \leftrightarrow |\beta|^2; \beta \to \beta^*} ;$$

$$D \cdot \rho_{01,12}^{(-)} = -(D \cdot \rho_{01,12}^{(+)}) \Big|_{\alpha^2 \leftrightarrow |\beta|^2},$$

$$D = 8\alpha \Omega_1 \Omega_2 \sqrt{\gamma_1 \gamma_2} \cdot [Im\beta \cdot (\Delta_1 - \Delta_2)(\gamma_1 + \gamma_2)(\Omega_1^2 + \Omega_2^2) -$$

$$2Re\beta \cdot \left((\Delta_1 - \Delta_2) \cdot (\Delta_2 \Omega_1^2 - \Delta_1 \Omega_2^2) + (\Omega_1^2 + \Omega_2^2)^2 \right)] +$$

$$(\alpha^6 + |\beta|^6 + 3\alpha^2 |\beta|^2) \cdot (\gamma_1 + \gamma_2)^2 \cdot (\Delta_1 - \Delta_2)^2 \cdot (\gamma_1 \Omega_2^2 + \gamma_2 \Omega_1^2) +$$

$$4\gamma_1 \Omega_2^2 \cdot [(\Delta_2 - \Delta_1)\Delta_1 + \Omega_2^2]^2 + 4\gamma_2 \Omega_1^2 \cdot [(\Delta_1 - \Delta_2)\Delta_2 + \Omega_1^2]^2 +$$

$$4\Omega_1^2 \Omega_2^2 [(\gamma_1 + 2\gamma_2)\Omega_1^2 + (\gamma_2 + 2\gamma_1)\Omega_2^2 + 2(\gamma_1 + \gamma_2)(\Delta_1 - \Delta_2)^2] .$$

$$p_+^{(st)} = \frac{|\beta|^2\gamma_2\Omega_1^2 + \alpha^2\gamma_1\Omega_2^2 - 2\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\alpha Re\beta}{\gamma_2\Omega_1^2 + \gamma_1\Omega_2^2 - 4\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\alpha Re\beta},$$

$$p_-^{(st)} = \frac{\alpha^2\gamma_2\Omega_1^2 + |\beta|^2\gamma_1\Omega_2^2 - 2\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\alpha Re\beta}{\gamma_2\Omega_1^2 + \gamma_1\Omega_2^2 - 4\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\alpha Re\beta},$$