Deep sub-Doppler cooling of Mg by light with elliptical polarization

Abstract: We study magneto-optical trap of ^{24}Mg atoms operating on the closed triplet $^3P_2 \rightarrow {}^3D_3$ (λ = 383.3 nm). We show the well-known light filed configuration does not allow to reach deep sub-Doppler cooling temperatures. It was considered a cooling in light field formed by light waves with elliptical polarization (ε-θ-ε^*configuration). This configuration offers 10 times lower cooling temperatures then conventional σ_+-σ_- MOT. Magnetic field and light field parameters for stable MOT working are discussed here.

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Institute of Laser Physics, Novosibirsk, 2016

Laser cooling of Mg

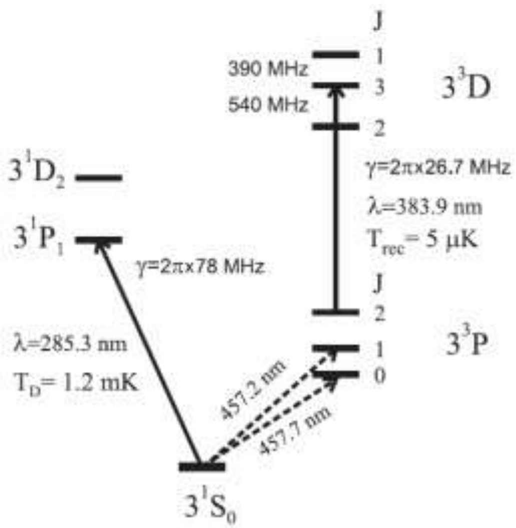

FIG. 1. Partial energy diagram of ^{24}Mg atom. Solid lines denote the cooling transitions with corresponding temperature limits, while dashed lines denote possible "clock" transitions, which can be used for laser stabilizing.

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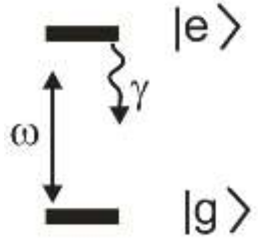
Atom	λ_{cl}	λ_m	BBR shift
Sr	698.5	813.5 [12]	-5.5×10^{-15} [41]
Yb	578.4	759.4 [21, 23]	-2.6×10^{-15} [41]
Ca	659.7	735.5 [24]	-2.6×10^{-15} [41]
Mg	**457.7**	$\approx$ **468** [40]	$\mathbf{-3.9\times10^{-16}}$ [41]
Hg	265.6	362.6 [26]	-2.4×10^{-16} [42, 43]

M. Riedmann, H. Kelkar, T. Wubbena, A. Pape, A. Kulosa, K. Zipfel, D. Fim, S. Ruhmann, J. Friebe, W. Ertmer, and E.Rasel, **PRA 86, 043416 (2012)**

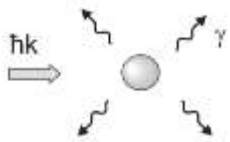
T-MOT temperature = 1mK above Doppler limit !

Basic mechanism of laser cooling

1) Two-level model



radiation pressure for moving atom

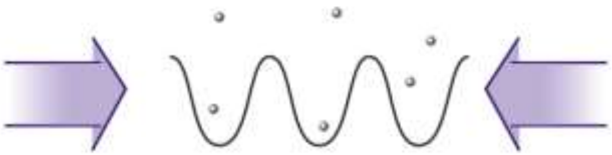


$F \approx \hbar k \gamma$

Doppler cooling

$$k_B T_D \approx \hbar\gamma / 2$$

dipole force

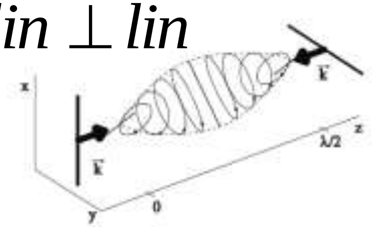


optical potential

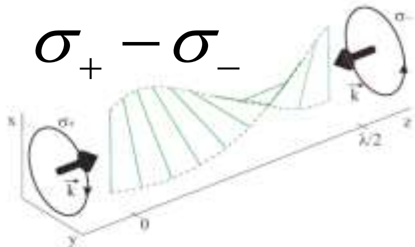
2) polarization effects

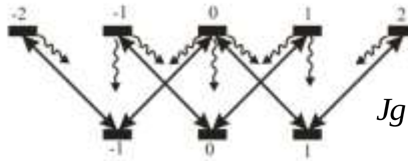
sub-doppler laser cooling

$lin \perp lin$



$\sigma_+ - \sigma_-$





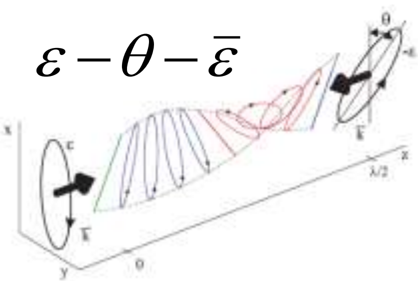
$Jg = 1 \rightarrow Je = 2$

J.Dalibard and C.Cohen-Tannoudji, J.Opt.Soc. Am. B (1989).

3) light fields with elliptical polarization

"anomalous" sub-Doppler cooling effects

$\varepsilon - \theta - \bar{\varepsilon}$



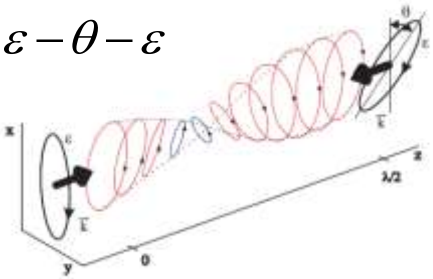
$jg = 1/2 \rightarrow je = 3/2$

$$\langle \xi_n \rangle = \hbar k^2 \tilde{\delta} \frac{3\sin^2\theta\cos^2(2\varepsilon_0)}{[1-\cos^2\theta\cos^2(2\varepsilon_0)]^{3/2}},$$

$$\langle \xi_a \rangle = -\hbar k^2 \frac{3}{2} \frac{\sin\theta\cos\theta\sin(2\varepsilon_0)\cos^2(2\varepsilon_0)}{[1-\cos^2\theta\cos^2(2\varepsilon_0)]^{3/2}}.$$

rectification of dipole force

$\varepsilon - \theta - \varepsilon$



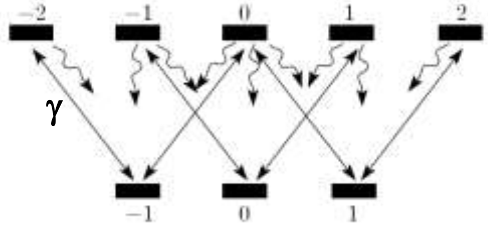
$\langle F \rangle \neq 0$

$\langle F \rangle(\varepsilon_0 = 0, \theta) = 0$

• O.N. Prudnikov, A. V. Taichenachev, A. M. Tumaikin and V. I. Yudin, *Kinetics of atoms in the field produced by elliptically polarized waves,* JETP vol.98, pp. 438-454, (2004)
• A.V. Bezverbnyi, O.N. Prudnikov, A.V. Taichenachev, A.M. Tumaikin, V.I. Yudin *The light pressure force and the friction and diffusion coefficients for atoms in a resonant nonuniformly polarized laser field* JETP v.96, pp. 383-401, (2003)
• O.N. Prudnikov, A.V. Taichenachev, A.M. Tumaikin, V.I. Yudin *Rectification of the dipole force in a monochromatic field created by elliptically polarized waves* JETP vol.93, pp.63-70 (2001)

Kinetics of atoms in light fields

example: optical transition 1 →2



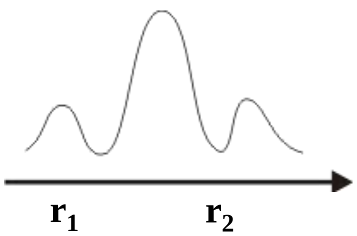
Two coordinate density matrix

$\hat{\rho}(z_1, z_2)$ Coordinate representation

$\hat{\rho}(p_1, p_2)$ Momentum representation

$$\hat{\rho}(r,p) = \int \frac{d^3 r}{(2\pi\hbar)^3} \hat{\rho}(r + q/2, r - q/2)\exp(-i\, p\cdot q/\hbar)$$

$$= \int \frac{d^3 u}{(2\pi\hbar)^3} \hat{\rho}(p + u/2, p - u/2)\exp(i\, r\cdot u/\hbar)$$



r = $(r_2 + r_1)/2$, q = $r_1 - r_2$

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}\right] - \hat{\Gamma}\{\hat{\rho}\} \qquad \hat{\rho} = \begin{pmatrix} \hat{\rho}^{ee} & \hat{\rho}^{eg} \\ \hat{\rho}^{ge} & \hat{\rho}^{gg} \end{pmatrix}$$

$$\frac{d}{dt}\hat{\rho}^{gg} = -\frac{i}{\hbar}\left[\hat{H}_{eff}, \hat{\rho}^{gg}\right] - \hat{\gamma}\{\hat{\rho}^{gg}\}$$ for $S = |\Omega|^2/(\gamma^2/2+\delta^2) << 1$

O. N. Prudnikov, et.al.,
JETP v.112, pp.939-945 (2011)

Semiclassical approach

Fokker-Plank equation

$$\left(\frac{\partial}{\partial t} + \sum_i \frac{p_i}{m}\nabla_i\right)W =$$

$$= -\sum_i \frac{\partial}{\partial p_i} F_i(\vec{r}, \vec{p})W + \sum_{ij} \frac{\partial^2}{\partial p_i\, \partial p_j} D_{ij}(\vec{r}, \vec{p})\, W$$

$$W(r,p) = Tr\{\hat{\rho}(r,p)\}$$

approximations: $\varepsilon_R = \frac{E_R}{\hbar\gamma} << 1$ $\quad \Delta p/\hbar k << 1$

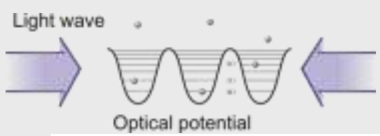
Quantum approaches

$\Psi \rightarrow \Psi'$

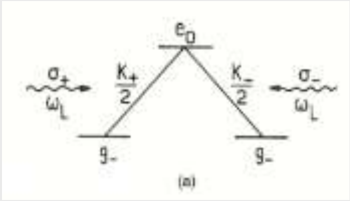
- quantum Monte-Carlo wave-function method
[J. Dalibard,et.al. PRL 68, 580 (1992)]

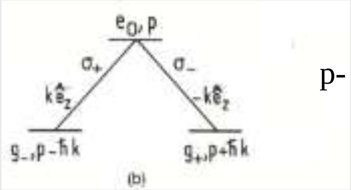
- Band theory (cooling in optical potential)
[Y. Castin,et.al. Europhys. Lett.14, 761 (1991)]

- cooling in σ_+ - σ_- field [A. Aspect,et.al. PRL **61**,826 (1988)].



$$\frac{\omega_{osc}}{\gamma_0} = \sqrt{\frac{27\hbar k^2 |\delta|}{M s_0 \Gamma^2}} = 6\frac{|\delta|}{\Gamma}\sqrt{\frac{E_R}{U_0}} \gg 1,$$





p- family approaches

Generalized continuous fraction method for density matrix equation

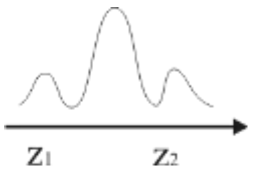
Atom-laser interaction part of Hamiltonian can be expressed as sum of two parts from opposite ligth waves.

$$\hat{V} = \hat{V}_1 e^{ikz} + \hat{V}_2 e^{-ikz} \qquad \hat{\rho}(z,q) = \sum_n \hat{\rho}^{(n)}(q) e^{ikz}$$

Density matrix equation in coordinate representation takes the following form for spatial harmonics $\rho^{(n)}$:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] - \hat{\Gamma}\{\hat{\rho}\} \quad \Longrightarrow \quad \frac{d}{dt}\hat{\rho}^{(n)} - n\frac{i}{M}\frac{\partial}{\partial q}\hat{\rho}^{(n)} = \hat{L}_0\left\{\hat{\rho}^{(n)}\right\} + \hat{L}_+\left\{\hat{\rho}^{(n-1)}\right\} + \hat{L}_-\left\{\hat{\rho}^{(n+1)}\right\}$$

1. We assume that the spatial coherence of density matrix is damped at enough large distance q_{max} we consider the spatial interval $[-q_{max}..q_{max}]$ and make a mesh with discrete points q_i (total N_q points). On the mesh we define derivative $\rho(n)$ in standard form:



$$\frac{\partial}{\partial q}\hat{\rho}_{q_i} \approx \frac{1}{2\Delta q}\left(\hat{\rho}_{q_{i+1}} - \hat{\rho}_{q_{i-1}}\right)$$

2. The equation can be written in Liouville representation with Liouvile operators L_0, L_+, L_- and density matrix in Liouville form:

$$\hat{\rho}_{q_i} \to \vec{\rho}_{q_i} = \begin{pmatrix} \cdot \\ \rho^{ee}_{\mu_e,\mu_e^{/};q_i} \\ \rho^{eg}_{\mu_e,\mu_g^{/};q_i} \\ \rho^{ge}_{m_g,\mu_e^{/};q_i} \\ \rho^{gg}_{m_g,m_g^{/};q_i} \\ \cdot \end{pmatrix}$$

$$\frac{d}{dt}\vec{\rho}^{(n)} + n\frac{i}{M}G\cdot\vec{\rho}^{(n)} = L_+\cdot\vec{\rho}^{(n-1)} + L_0\cdot\vec{\rho}^{(n)} + L_-\cdot\vec{\rho}^{(n+1)}$$

Example: For the case of optical transition 1/2→3/2 the vector ρ contains 18 x Nq elements, for the case of 1→ 2 it contains 34 x Nq elements.

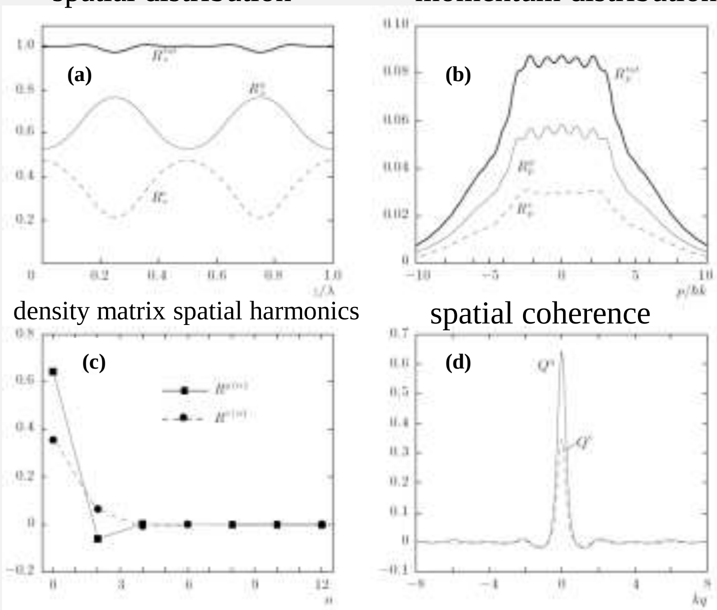
O.N. Prudnikov, A.V. Taichenachev, A.V. Tumaikin, V.I. Yudin, JETP v. 104, p839, (2007)
O. N. Prudnikov, R.Ya. Ilenkov, A. V. Taichenachev, A. M. Tumikin, and V. I. Yudin, JETP v.112, p.939 (2011)

Solution of density matrix equation

Example 1

spatial distribution — momentum distribution

density matrix spatial harmonics — spatial coherence

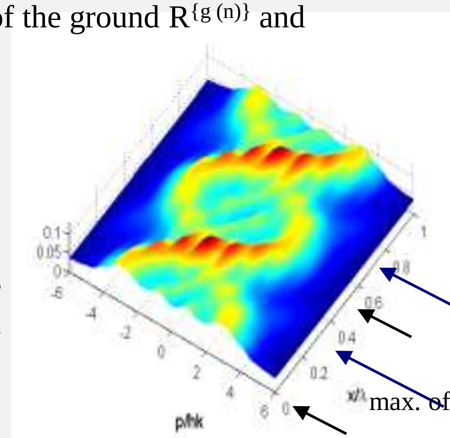


Density matrix spatial (a) and momentum (b) distributions for atoms with Jg=1/2→Je=3/2 optical transition in standing wave with linear polarization (δ= - γ/2, Ω = γ, ω_R = 0.1γ).
Density matrix spatial harmonics of the ground $R^{\{g\,(n)\}}$ and excited state $R^{\{e\,(n)\}}$ (c).

$$R^{e(n)} = Tr\left\{\hat{\rho}^{ee(n)}(q=0)\right\}$$
$$R^{g(n)} = Tr\left\{\hat{\rho}^{gg(n)}(q=0)\right\}$$

Spatial coherence functions for the ground Q^g and excited state Q^e (d).

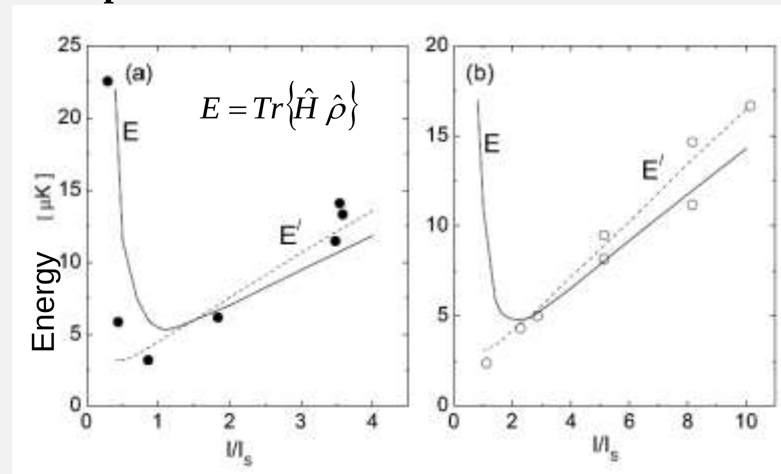
$$Q^e(q) = Tr\left\{\hat{\rho}^{ee(n=0)}(q)\right\}$$
$$Q^g(q) = Tr\left\{\hat{\rho}^{gg(n=0)}(q)\right\}$$



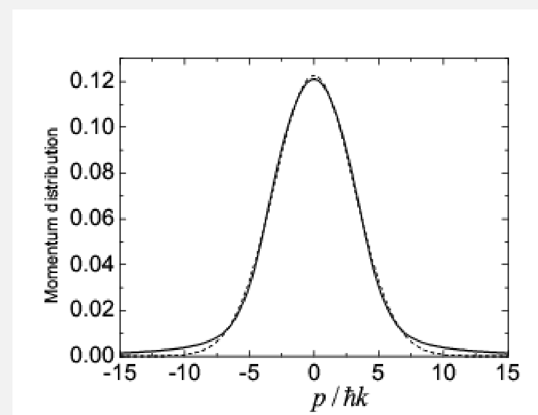
max. of U

min. of U

Example 2



Energy of [85]Rb atoms in lin⊥lin field ($5S_{1/2}$ (F = 3)→$5P_{3/2}$ (F'=4)).
The black and white dots represents the temperature measurements results [P. S. Jessen, et.al., Phys. Rev. Lett. **69,** 49 (1992)]



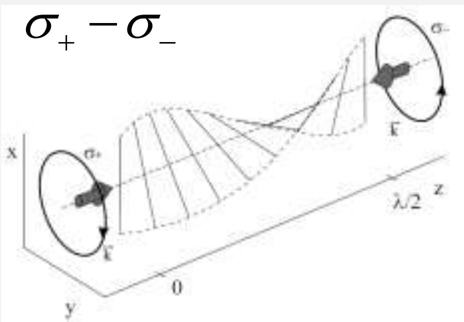
Momentum distribution (solid line), and Gaussian approximation (dashed line).
I/Is = 1.4, δ = -8γ

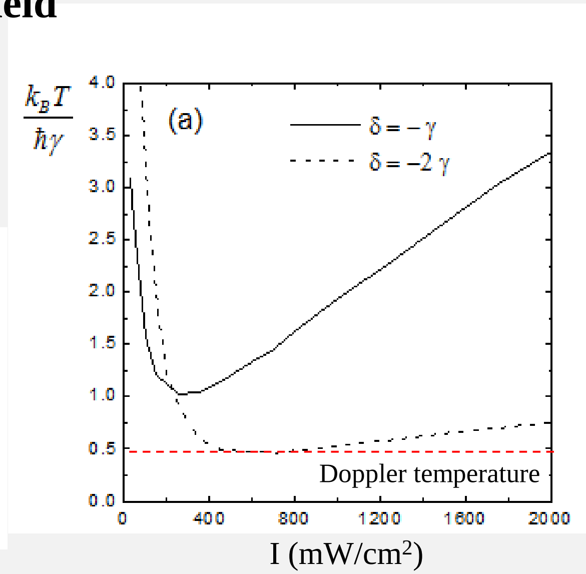
$$F(p) \approx e^{-\frac{p^2}{2mT}}$$

Sub-Doppler cooling of ^{24}Mg: quantum approach

laser cooling in σ_+-σ_- field

$$T = \hbar\gamma \approx 1.28\, mK$$

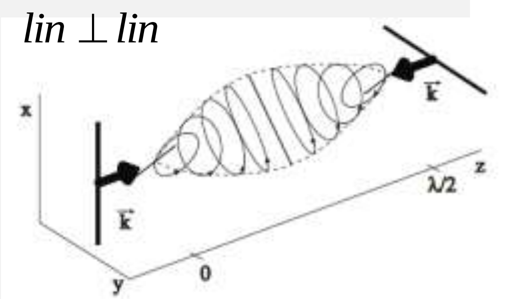


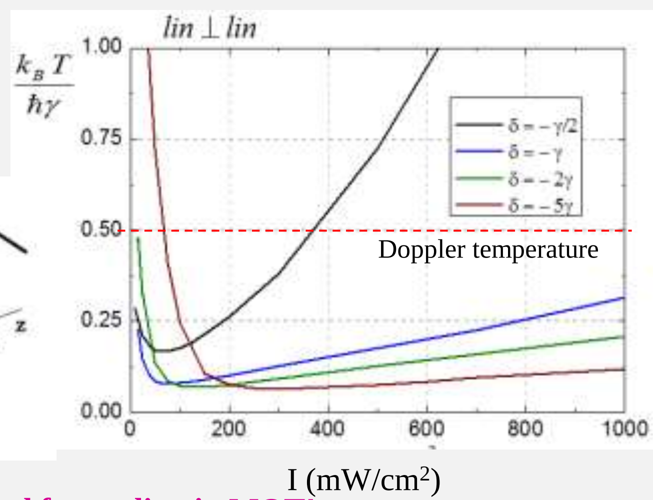


δ/γ	$k_B T\ [\hbar\gamma]$	I (mW/cm²)
-0.5	1.53	250
-1	1.02	260
-2	0.46	700
-5	0.23	1000

Fig. Optimal parameters of intensity and minimum temperature for cooling in σ_+-σ_- field

laser cooling in lin⊥lin field





δ/γ	$k_B T\ [\hbar\gamma]$	I (mW/cm²)
-0.5	0.167	70
-1	0.078	80
-2	0.070	150
-5	0.064	300

Fig. Optimal parameters of intensity and minimum temperature for cooling in lin⊥lin field

lin⊥lin configuration can't be used for cooling in MOT!

Sub-Doppler cooling conditions

for $S = |\Omega|^2/(\gamma^2/2+\delta^2) << 1$

Sub-Doppler temperature

$$T = \frac{\langle p^2 \rangle}{M} = -\frac{\langle D^{(0)} \rangle_z}{\langle \xi_s \rangle_z} = \hbar\gamma\,\beta(\delta/\gamma)\,S$$

slow atom approach:

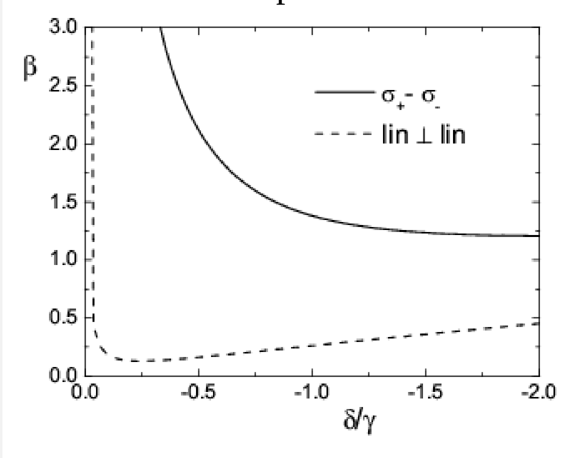
$$p < \gamma M S / k$$

$$\frac{\hbar k^2}{M\gamma}\beta(\delta/\gamma) < S$$

$$\varepsilon_R = \frac{\hbar k^2}{\gamma 2M}$$

negligible in semiclassical approach

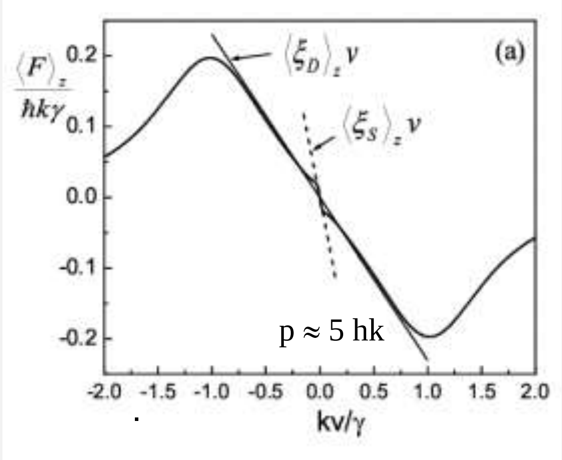
β(δ/γ) function for atoms with 2→3 optical transition



What happens for non negligible ε_R ?

$$\varepsilon_R = \frac{1}{2\tilde{M}}$$

	optical transition	Is [mW/cm²]	λ [nm]	$\tilde{M}$
^{7}Li	$2\,^2S_{1/2} \to 2\,^2P_{3/2}$	2.56	671	46
^{23}Na	$3\,^2S_{1/2} \to 3\,^2P_{3/2}$	6.34	589	198
^{39}K	$4\,^2S_{1/2} \to 4\,^2P_{3/2}$	1.81	766	358
^{85}Rb	$5\,^2S_{1/2} \to 5\,^2P_{3/2}$	1.63	780	770
^{133}Cs	$6\,^2S_{1/2} \to 6\,^2P_{3/2}$	1.06	852.3	1270
^{52}Cr	$4\,^7S_3 \to 4\,^7P_4$	8.49	425.6	115
^{27}Al	$3p\,^2P_{3/2} \to 3d\,^2D_{5/2}$	57	309.4	85
^{69}Ga	$4p\,^2P_{3/2} \to 4d\,^2D_{5/2}$	127	294.4	382
^{115}In	$5p\,^2P_{3/2} \to 5d\,^2D_{5/2}$	78	325.7	634
^{107}Ag	$5\,^2S_{1/2} \to 5\,^2P_{3/2}$	76.8	328	601
^{24}Mg	$3^3P_2 \to 3^3D_3$	61.7	383.9	238



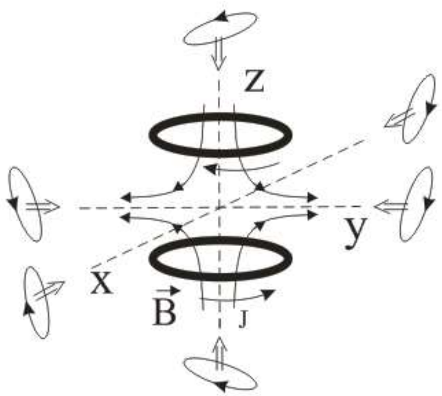
1) violation of slow atom approach
2) range of sub-Doppler force might be few recoil momentum.

atom may not "see" sub-Doppler force!

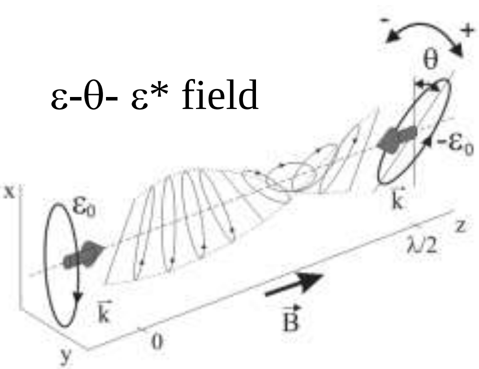
Sub-Doppler cooling of ^{24}Mg in MOT

No trapping force in lin⊥lin filed!

$\hbar\gamma \approx 1.28\,mK$



ε-θ- ε* field



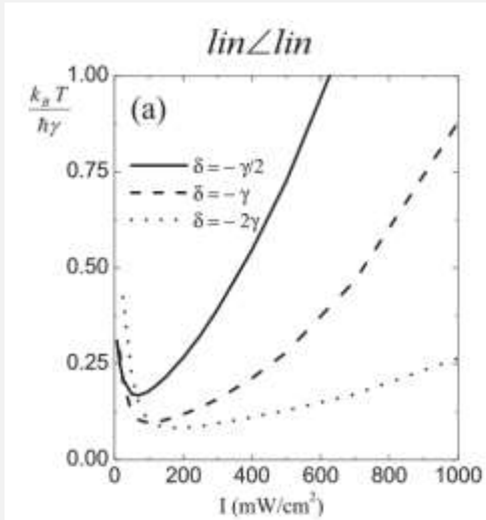


Fig. Temperature of laser cooling of ^{24}Mg in lin-θ-lin field (θ = - π/4)

δ/γ	$k_B T\ [\hbar\gamma]$	I (mW/cm²)
-0.5	0.168	70
-1	0.097	100
-2	0.083	200
-5	0.080	400

Fig. Optimal intensity and minimum temperature in lin-θ-lin field (θ = - π/4)

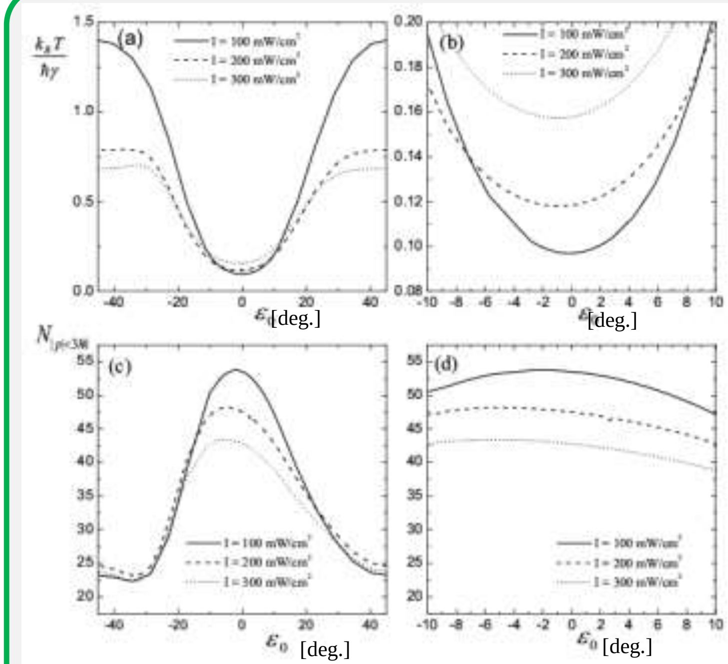


Fig. Temperature of laser cooling of ^{24}Mg in ε-θ-ε* field (θ = - π/4, δ = -γ) and fraction of atoms with |p|<3hk.

I [mW/cm²]	100	200	300
ε_0 [deg.]	-0.28	-0.86	-1.15
T [hγ]	0.097	0.118	0.157
ε_0 [deg.]	-2	-5.16	-5.16
$N_{\|p\|<3hk}$ [%]	53.7	48.2	43.4

Fig. Optimal ellipticity and minimum temperature for different intensity in ε-θ-ε field (θ = - π/4).

Magneto-optical potential for ^{24}Mg in in ε-θ-ε* MOT

assuming linear growth of magnetic field in trapping zone

$$U^{(H)} = -\frac{R_W}{\Omega_H(R_W)} \int_0^{\Omega_H(R_W)} \left\langle F^{(H)}(v=0,\Omega_H) \right\rangle d\Omega_H$$

$\Omega_H/\gamma = 1$ (H = 12.7 Gs)

$\sigma_+ - \sigma_-$

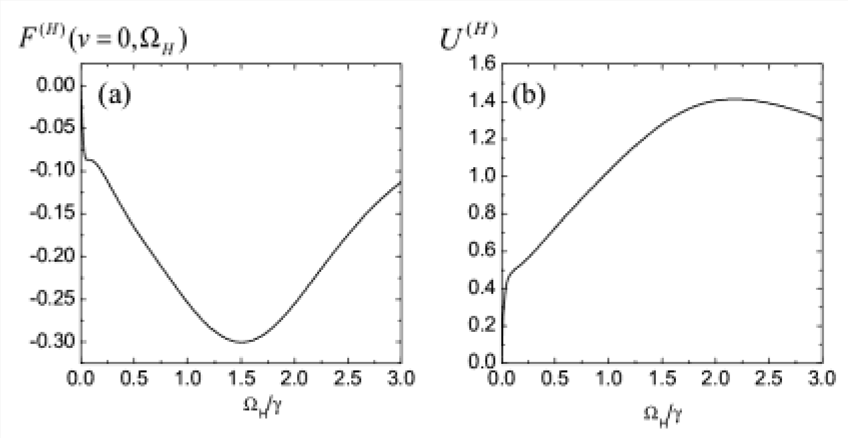


Fig. Force in hγ units and magneto-optical potential in $h\gamma R_W/\lambda$ units (R_W is beam radius) as function of Zeeman splitting on MOT edge in σ_+-σ_- MOT. (I = 100mW/cm^2, δ = -γ)

R_w is radius of light beams forming the MOT

ε-θ-ε*

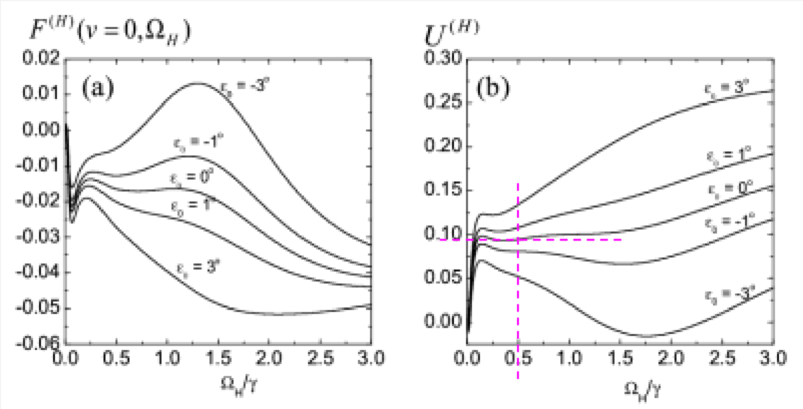


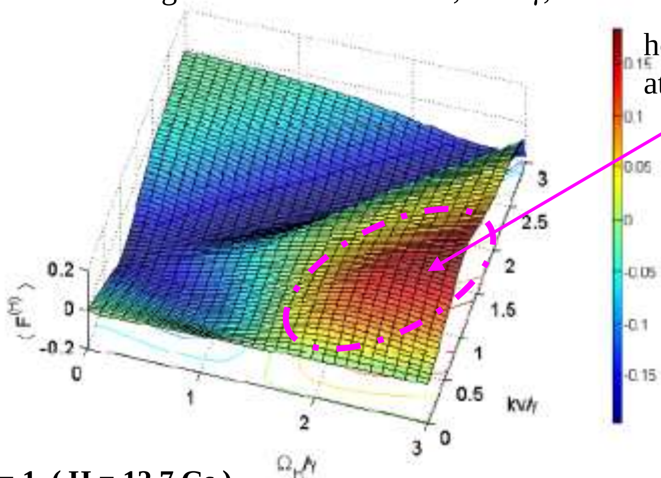
Fig. Force in hγ units and magneto-optical potential in $h\gamma R_W/\lambda$ units (R_W is beam radius) as function of Zeeman splitting on MOT edge in ε-θ-ε* MOT. (I = 100mW/cm^2, δ = -γ)

Estimation: R_W = 0.5 cm,
for magnetic field gradient ∂_z H = 12.7 Gs/cm
(Ω_H/γ = 0.5 Gs at edge),
the MOT depth $U^{(H)}$ = 0.094 $h\gamma R_W/\lambda \approx$ 1.56K that much exceed sub-Doppler cooling temperature T≈125 μK.

Magneto-optical force for trapping in ε-θ-ε* MOT

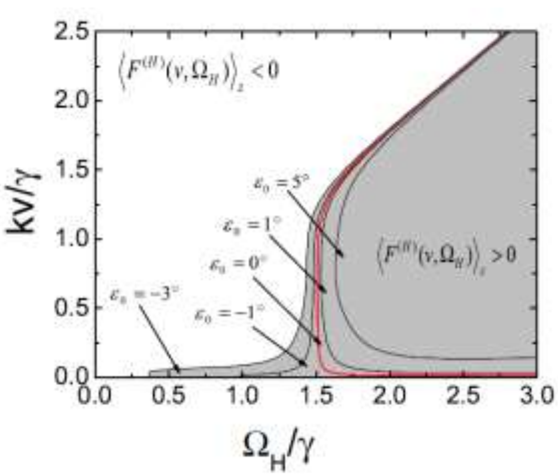
Trap should be stable for moving atom as well.

Magneto-optical force as function of atoms velocity and magnetic field.
(lin-θ-lin field configuration with $\theta = -\pi/4$, $\delta = -\gamma$, $I = 100$ mW/cm^2)



$\Omega_H/\gamma = 1$ (H = 12.7 Gs)

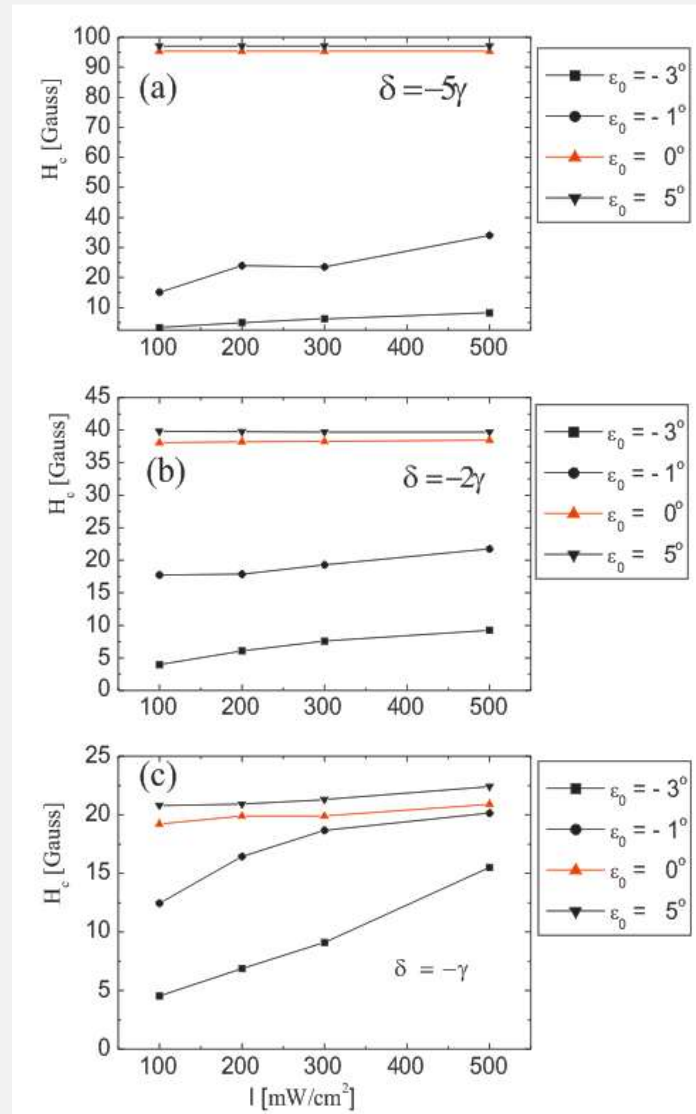
Magneto-optical force trapping zone for ε -θ-ε* MOT ($\theta = -\pi/4$, $\delta = -\gamma$, I = 100 mW/cm2)



Number of trapped atom $N_c \sim v_c^4$
[C. Monroe, et.al. PRL. 65, 1571 (1990)]

$v_c > 3.5k/\gamma$ results $N_c \sim 10^7 - 10^8$ at.

Critical magnetic field for stable ε-θ-ε* MOT



Results

1. Quantum recoil effects resulting sub-Doppler cooling are not efficient for cooling of ^{24}Mg on $^3P_2$-$^3D_3$ in σ_+-σ_- field configuration.

2. For deep sub-Doppler cooling lin⊥lin configuration can be used.
(Can't be used for MOT!)

3. For deep sup-Doppler cooling in MOT ε-θ-ε* light field configuration can be used.

- enough deep magneto-optical potential

- for stable MOT magnetic field should not exceed critical values in trapping zone.

- positive small ellipticities ($\varepsilon < 5$ deg.) of light waves forming the MOT are preferred to increase much the critical values for magnetic field.

Ellipticity parameter